Volume 2 Existence — reading the published construction

Part 6 What has been proved

Kiichi, Shiori and Rin · Edition of 2026-11-14 · https://computoergosum.com/en/principia/yang-mills/

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Entrance — the promise of this volume

> This chapter is a guide.

What this chapter shows — What Volume 2 reads (the published Lean code and the committee's assessment). How sources are cited. The meaning of the Volume 2 labels Source Inferred Assessment , and how they relate to the labels of Volumes 1 and 3. The level of certainty "the committee's assessment is that it is proved in Lean", and where the line falls for what this book has checked itself. For each part, how far the reading has got.

Prerequisite chapters — 0-03 Reading the problem statement word by word, 4-09 The axioms of quantum field theory, 4-10 The Osterwalder–Schrader axioms and reconstruction. For the historical background of constructions, 7-07 Four dimensions.

Said first — This book has neither built nor run the published Lean code. Whether the proof is correct is quoted at the level of the committee's assessment. What this book has checked itself goes only as far as reading the definitions and the form of the statements in the source. The published construction does not show the mass gap (6-04).

Order of this chapter

1. What this volume reads — two primary sources, and the committee
2. Levels of certainty — three levels, and what this book does not do
3. Key to the labels — Source Inferred Assessment , and how they relate to the labels of Volumes 1 and 3
4. How we cite — how locations are written, not copying, names
5. Which target a statement belongs to — five distinctions, and the meaning of "used"
6. How deep the reading goes — for each part, what has been read and what has not
7. The shape of the chapters of Volume 2, and the chapters of this part — the four-section shape, and the five chapters of Part 6

01

What this volume reads

On 10 November 2026, code written in Lean 4 for a construction establishing the existence of four-dimensional pure Yang–Mills theory was published, together with the assessment of a committee that examined it [Assessment] . Volume 2 reads these two as primary sources.

Source	Contents	Version
Code `yang-mills-group/OpenAI-YM-20261028` (Apache-2.0)	The Lean 4 source submitted by OpenAI on 2026-10-28. 9,477 files, about 990,000 lines [Source]	3456e5e
Assessment `docs/assessment-1.pdf` in `yang-mills-group/YM`	The committee's assessment (35 pages, 2026-11-10). Part I compares the results of the construction with the problem statement; Part II is the record of the checks	55ba9fc
The committee's statement	The third statement on the page of the Kavli Institute for Theoretical Physics (2026-11-09). Signed by Michael R. Douglas, David J. Gross (chair), Arthur Jaffe, Igor R. Klebanov and Nikita Nekrasov	—

The text of the assessment does not name the committee members; the names come from the signatures on the statement. The verdict at the opening of the assessment is as follows [Assessment] . The submitted code proves, for every simple compact group, that a relativistic quantum field theory satisfying the Wightman axioms can be built as the scaling limit of four-dimensional Wilson lattice gauge theory. It does not prove that the infinite-volume limit is unique, or that there is a mass gap.

The assessment also says the following about itself.

- Only the Lean source was used for the assessment; a separate manuscript of a paper proof was not used [Assessment] .
- To help read the Lean code, OpenAI's model Astra was used [Assessment] .
- The committee does not claim yet to understand the proof fully ("We do not claim to fully understand the proof yet.") [Assessment] .

This book takes over this position of the committee as it is. When Volume 2 writes "what has been proved", it means "what the committee has assessed as such".

02

Levels of certainty

Each statement in Volume 2 sits at one of the following three levels. The upper two are on the committee's side; the third is on this book's side.

Level	Contents	Who checked	What this level does not guarantee
Accepted by the Lean kernel	Rebuilding, auditing the axioms, re-checking in the kernel, and matching statements and definitions with Comparator all passed (assessment §9.1, §9.2)	The committee Assessment	That the definitions express the intent of the problem
The statements express the intent of the problem	A human reading of the definitions (the assessment describes it as "extensive, but not exhaustive") and a comparison with independently written axioms (§9.3, §10, §11)	The committee Assessment	That a human has fully understood the proof
The form of the statements and definitions has been read	The source files were opened, and the names, lines, order of quantifiers and hypotheses of the declarations were read	This book Source	That the content of the proof is correct

We state here, once and plainly, what this book **does not do**.

- It has not built the published Lean code. Nor has it run it.
- It has not repeated the re-checking in the kernel or the Comparator check on its own machine.
- Of the content of the proof (the sequences of tactics and the proofs of lemmas), it has read only part of the outline. Where the reading falls short is written in the last section of each chapter.

Volume 2 therefore writes every result of the published construction at the level "the committee's assessment is that it is proved in Lean". What this book can say goes only as far as: the statement is written in the source, at that location, in that form.

03

Key to the labels

The labels specific to Volume 2 are the following three. Each sentence carries one label.

Label	Meaning	What accompanies it	What it does not guarantee
Source	The source of the published construction (the wording of definitions and statements), or the source on the assessment side (the code and examples of the committee's checks), has been read	The relative path of the file, the name of the declaration, the line	That the proof is correct
Inferred	This book's inference (from names, explanatory text, the outline of the proof)	What it was inferred from	That the source says so
Assessment	Based on the committee's assessment	The section of the assessment	That this book has checked it

A Source that needs a qualification carries it in brackets, as in "Source (within the scope of a name search)". When we write "not in the source", we add the words searched for and the scope.

We note how these relate to the labels of Volume 1 (Parts 0–5) and Volume 3 (Parts 16–28).

- In the text of Volume 2 too, the labels of the whole book are used, with the same meaning as in each volume: the Volume 3 labels Proved here Cited Computer-assisted Reading Assumption and Lean , and in parts of Parts 7 and 15 the Volume 1 labels (Lean , Paper , Computed , Known , Physics). The meanings of the labels are in "Labels" in 0-01.
- Cited is used only for known results checked in the literature (the Osterwalder–Schrader reconstruction, the Arzelà–Ascoli theorem and the like). Sentences about the theorems and definitions of the published construction carry Source , and sentences touching the level "it has been proved" carry Assessment . Cited and Source are not put side by side on one sentence.
- Reading is used only in the sense of "reading or observing numbers" (floating point, Monte Carlo, fits); having read the source is expressed by Source .
- In the "type" column of the source tables, bibliographic items such as the problem statement and the statement of the committee are marked "Cited".
- When the section "What this book's investigation had seen" of Volume 2 quotes this book's Lean theorems, it writes them as Volume 3 does (the theorem's name and its scope on the same line).

04

How we cite

There are two kinds of source.

- **Code**: the relative path of the file, the name of the declaration, the line. Example: `JointExistence` in `ChallengeSpec.lean` (line 186). Unless stated otherwise, paths are relative to the root of the repository `OpenAI-YM-20261028`. The code of the committee's checks is written relative to `submissions/openai-20261028/` in the repository `YM`.
- **Assessment**: the section number. Example: assessment § 5. The glossary in the appendix is "assessment Appendix A".

We do not copy long passages of the published construction's text or code. We write the mathematics in our own words. Lean statements are shown as they are only where the form itself matters, and then for a few lines at most (such as the public theorem in 6-02).

The conventions for names are as follows.

- Objects of the published construction are called by **names that say what they mean** (the probability kernel, the predictor, the truncated action, forced activities, the induction hypothesis, and so on). The identifiers are added in monospace as sources.
- Many of the locations and declarations in the code have names unconnected with the mathematical content. The assessment also writes that the naming departs from convention (§ 9.1) `Assessment` . Such names are written only as locations, and are not used as names in the running text.
- Line numbers refer to the versions above.

05

Which target a statement belongs to

The published code contains several "main theorems" of different strength. Which one a statement belongs to changes how strongly it may be quoted. Volume 2 distinguishes the following five.

Distinction	Contents	Location
The public theorem	The statement in the form of the published question. Its only hypothesis is "the Lie algebra of the group is simple"	`JointExistence` in `ChallengeSpec.lean`, proved by `jointExistence` in `Solution.lean`
The construction theorem	A stronger statement that contains the public theorem as a projection	`constructed` in `Construction.lean`

Distinction	Contents	Location
The committee's combined target	A statement in which the committee bundles, over a single family, its independent checks on top of the construction theorem	`combined_acceptance` in `submissions/openai-20261028/checks/CombinedAcceptance.lean` of `YM`
Reached from one of the headline theorems	In the dependencies of one of the 17 headline theorems listed by the verification record (the status in the table is headline for the headline theorems themselves, used for dependencies)	`verification/joint-headlines.txt`, `declaration-usage.tsv`
Outside the dependencies	Not reached from any of the 17 headlines	Status outside in the same table

One caution is needed. The status in the table of the verification record, `verification/declaration-usage.tsv`, takes three values: headline (the 17 headlines themselves), used (100,151) and outside (150,151) Source . "Used" means reached from **one** of the 17 headline theorems (`verification/Usage.lean`, lines 17–40) Source . Besides the public theorem and the construction theorem, the 17 include headlines on other subjects Source . So "used" does not necessarily mean "used by the public theorem".

In Volume 2 we write "used in the proof of the main theorem" only for declarations whose chain of dependencies we have checked link by link up to the public theorem or the construction theorem. What has not been checked is written as "not checked". As an example, take the short-distance effective coupling (Theorem 7.2 of assessment § 7.4). The assessment does not name its location in Lean, and the source contains three things of a similar form Source .

- `YM.PertExtension.Running.ratio` (`YM/PertExtension/Running.lean:145`): a statement in the form of a ratio; status outside (outside the dependencies).
- `YM.Trace.Pilot.wilson_link` (`YM/TraceExtension/Atlas/Sink.lean:50`): status used. The first headline is `continuum_with_position_tests`, the direct user is `crest_ready`, and it is used as an intermediate step in the analysis of the trace. This is the one whose wording is close to the statement of Theorem 7.2 Inferred . Its conclusion does not survive into the conclusion of the public theorem, the construction theorem or the committee's combined target Source .
- `YM.Pert.Asymptotics.comparison` (`YM/Pert/Asymptotics/RawAxis.lean:36`): a headline theorem itself (headline). It is the running of the coupling $\beta(N_i, k_i, i)^{-1/2}$ at step i with a cutoff, a different quantity from gW.

This is treated in detail in 6-02 and 13-03.

06

How deep the reading goes

For each part, we list the range read in the source and the main places not yet read. The "What has not been read" section at the end of each chapter gives this in detail.

Part	What has been read Source	Main places not yet read
Part 6 What has been proved	The full text of the public theorem, the statement of the construction theorem, the chain of main theorems, the statement of the committee's combined target, the summary of the verification record	The proofs in each file of the committee's independent checks
Part 8 From the lattice to the continuum — objects	The definitions of the Wilson family, of correlations and of the order of limits; the definitions of field labels and quotients	The content of the inverse of the renormalisation transport (how it mixes with the identity field)
Part 9 One step of renormalisation	The probability kernel, patches and weights, the predictor, the branching of gauge fixing, the statements of symmetry	The details of the proof of the averaging that makes a symmetric gauge fixing
Part 10 Matching the coupling, and asymptotic freedom	The calendar, the shooting band, the statement of the difference of couplings in one step	The explicit form of constants satisfying the conditions of the calendar
Part 11 Multistep estimates and large fields	The form of the induction hypothesis and of the one-step certificate, the definitions of large-field regions and seeds, the statements of the one-step inequalities	The core of the proof of the one-step theorem, the reorganisation of activities after one step, the lemma on the Haar loss
Part 12 Infinite volume and reconstruction	Reflection positivity, the chessboard inequality, the input from the ultraviolet, the Arzelà–Ascoli extraction, the phase decomposition, the statement of reconstruction	The body of the estimates on the transfer operator, the outline of the proof that the algebra of phases is commutative
Part 13 Evidence that it is Yang–Mills	The statements of the stress tensor, the Ward identities, the trace law and the operator product expansion	The content of some fields of the strong forms concerning the trace, the location of the second coupling

On the whole, what this book has been able to read goes as far as "what is stated, where, and in what form" and "in what order it is assembled". Of the multistep estimates (Part 11), which take up about 70% of the code, only the form of the statements and the outline of some lemmas have been read Inferred .

07

The shape of the chapters of Volume 2, and the chapters of this part

The chapters of Parts 8–13 have the following four sections, in this order.

1. **Statements** — what the published construction defines and states on the subject, with sources.
2. **Outline of the mechanism** — in what order, and using what, it is assembled. With formulas.
3. **What this book's investigation had seen** — how far this book's research and Lean had got on the subject. If nothing, "nothing".
4. **What has not been read** — what has not been read and what has not been checked, with locations.

At the end of each chapter, as in Volumes 1 and 3, there are the tables "What this chapter can claim" and "Sources and reproduction". Part 14 is the part of comparison, and Part 15 is the record of this book's attempts at the ultraviolet.

This part (Part 6) is the part of the entrance and the overview. It does not follow the four-section shape as it is, but each chapter has the sections "What this book's investigation had seen" and "What has not been read".

Chapter	In one line
6-01 The four requirements of the problem statement, and the published construction	Of the four — axioms, local fields, short distances, mass gap — what has been reached and what has not.
6-02 Reading the statements	The difference in scope between the public theorem, the construction theorem and the committee's combined target. Who chooses what. What the verification record has checked.
6-03 The skeleton of the proof	The ten stages from the calendar to the evidence, in one picture and one table. A guide to the chapters of Volume 2 that treat each stage, and the size of each layer of the code.
6-04 What is not shown	Convergence and uniqueness, a probability measure, the mass gap, Wilson loops, one-particle states and more. A bridge to Volume 3.

What this chapter can claim

Said	Not said
Assessment The committee judged that the submitted code proves the construction of a theory satisfying the Wightman axioms (opening of the assessment)	The committee itself does not say that it fully understands the proof. Uniqueness and the mass gap have not been shown
Source The statements Volume 2 quotes are written in the source, at those locations, in that form	The correctness of the content of the proofs (this book has neither built nor run the code)

Said	Not said
The conventions (labels, levels, citation, distinctions of targets)	— (these are conventions, not mathematical statements)

Sources and reproduction

Item	Type	Source, tool
The published code	Source	yang-mills-group/OpenAI-YM-20261028 (Apache-2.0, submitted 2026-10-28, version 3456e5e). The numbers of files and lines were counted on this machine
The assessment	Assessment	docs/assessment-1.pdf in yang-mills-group/YM (2026-11-10, 35 pages, version 55ba9fc). Opening, § 9, § 12. The committee members are not named in the text
The committee's statement	Cited	The third statement on the page of the Kavli Institute for Theoretical Physics (2026-11-09). Signed by Michael R. Douglas, David J. Gross (chair), Arthur Jaffe, Igor R. Klebanov and Nikita Nekrasov
The meaning of "used" and the three values of status	Source	`verification/Usage.lean`, lines 17–40; `verification/joint-headlines.txt` (17 lines); `verification/declaration-usage.tsv`
The three locations of the short-distance effective coupling	Source	`YM/PertExtension/Running.lean:145`, `YM/TraceExtension/Atlas/Sink.lean:50`, `YM/Pert/Asymptotics/RawAxis.lean:36`
The problem statement	Cited	The problem statement of Jaffe–Witten. For how to read it, 0-03

Terms used in this chapter — the assessment, the committee's combined target, the public theorem, the construction theorem, headline theorems, outside the dependencies, Comparator, re-checking in the kernel. Their meanings are in A-02 Glossary.

Next chapter to read: 6-01 The four requirements of the problem statement, and the published construction — the four things the problem statement asks for, and where the published construction has got to, side by side in one table.

2026-11-13 · chapter Yang–Mills and the mass gap Part 6 What has been proved problem statement and construction · Kiichi, Shiori and Rin

The four requirements of the problem statement, and the published construction

> This chapter is established knowledge (a reading of the published construction). § 05 is this book's investigation.

What this chapter shows — The four requirements that the assessment extracts from the problem statement. How far the published construction reaches on each of them (what is reached, what is not, the location in Lean, the section of the assessment). The standard of "existence" the committee adopted, and the reservations the committee itself writes. Where among the four the results of Volume 3 of this book lie.

Prerequisite chapters — 6-00 Entrance (labels and levels of certainty), 0-03 Reading the problem statement word by word, 4-09 The axioms of quantum field theory, 4-06 Asymptotic freedom.

Said first — The wording of the four requirements is the summary in § 1 of the assessment, not the original text of the problem statement itself. The published construction claims nothing about the fourth, the mass gap. Every "reached" in this chapter is at the level "the committee's assessment is that it is proved in Lean".

Order of this chapter

1. The four requirements of the problem statement — the summary in § 1 of the assessment
2. What the published construction gives — the verdict at the opening of the assessment, and § 2
3. Requirement by requirement — what is reached, what is not, locations (Table 1)
4. The committee's standard of "existence", and its reservations — subsequences, the vacuum, the degree of understanding
5. What this book's investigation had seen — where among the four this book's results lie (Table 2)

6. What has not been read — comparison with the original text, statements whose location is unknown

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Labels: Source = the source of the published construction has been read / Inferred = this book's inference / Assessment = based on the committee's assessment. The other labels (Proved here, Cited, Reading, Lean and so on) have the same meaning as in the rest of the book (for details, "Labels" in 6-00).

01

The four requirements of the problem statement

The introduction is 0-03. Assessment § 1 summarises what the Jaffe–Witten problem statement asks of a constructed Yang–Mills theory as the following four Assessment .

1. **A non-trivial theory satisfying the axioms.** Positivity, spacetime symmetry, locality, regularity and the representation of the vacuum hold at the strength of the Wightman or Osterwalder–Schrader (OS) framework, with genuine local observables.
2. **The local fields of Yang–Mills.** The space of operators corresponds to the gauge-invariant polynomials in the curvature F and its covariant derivatives.
3. **The predicted short-distance behaviour.** Asymptotic freedom and the theory of perturbative renormalisation, the stress tensor, and an operator product expansion with the predicted singularities.
4. **A positive mass gap.** For the non-negative Hamiltonian H, the following holds for some $\Delta > 0$.

$$\sigma(H) \subset \{0\} \cup [\Delta, \infty). \quad (1)$$

For the fourth, the assessment adds "that the supremum of such gaps is finite" Assessment . The Δ of (1) is the mass gap. It means that the energies of states orthogonal to the zero-energy space are separated by at least Δ. That the zero-energy space consists only of multiples of the vacuum (the uniqueness of the vacuum) is an axiom separate from (1). In Euclidean language it corresponds to the exponential decay of correlations (4-10).

02

What the published construction gives

We write the verdict at the opening of the assessment once more [Assessment] . The submitted Lean code proves, for every simple compact group, that a relativistic quantum field theory satisfying the Wightman axioms can be built as the scaling limit of four-dimensional Wilson lattice gauge theory. It does not prove that the infinite-volume limit is unique, or that there is a mass gap.

Assessment § 2 divides what the construction gives into four [Assessment] .

- Starting from Wilson's lattice gauge theory, it associates renormalised lattice observables with polynomials in the curvature, and shows that their correlations converge to a limit that defines a Euclidean field theory. From there, by OS reconstruction, it builds a theory satisfying the construction's own version of the Wightman axioms. It also shows that the theory is non-trivial.
- It defines renormalised local operators corresponding to polynomials in the curvature, including a conserved stress tensor. There are several short-distance checks.
- It gives an operator product expansion with error bounds, but does not show agreement with the predictions of the perturbative expansion. It gives the scaling behaviour of the coupling needed for asymptotic freedom. It defines the trace of the stress tensor and shows its properties.
- **On the mass gap, it gives no evidence, either for or against.**

These four correspond almost one to one with the four requirements of § 01. In the next section we list the locations in the source requirement by requirement.

03

Requirement by requirement

What the verdict at the opening of the assessment says is "proved" is the construction of a theory satisfying the Wightman axioms, which corresponds to requirement 1 [Assessment] . § 7 states the individual short-distance results (the short-distance law of the trace, the effective coupling, the ratio of two couplings, the comparison with the free Maxwell field) as theorems the submission proves ("The submission proves that ...", § 7.2, § 7.4) [Assessment] . The assessment then calls them together "substantial short-distance evidence at the level of the leading terms", and, together with the lattice origin, the local fields, the stress tensor and the operator product expansion, substantial evidence that the construction describes Yang–Mills (end of § 7.4, opening) [Assessment] . For requirements 2 and 3, the assessment lists the definition of the local operators and "several short-distance checks", calls them together

"substantial evidence that it is Yang–Mills", and states that agreement with perturbation theory at all orders has not been proved (§ 2, § 7, § 9.4) Assessment. Please read rows 2 and 3 of Table 1 in this light. The "Location" column of Table 1 gives the fields in the statement of the public theorem (`ChallengeSpec.lean`). The differences between the public theorem, the construction theorem and the committee's combined target are treated in 6-02.

Table 1 The four requirements of the problem statement, and the published construction (labels are per clain row)

Requirement	What the construction gives	What is not reached	Location, assessme
1 Axioms and non-triviality	Assessment The infinite-volume theory is decomposed into phases, and each component (phase) satisfies a variant of the OS axioms including clustering. A Wightman theory is reconstructed from each component. In a chosen phase the two-point function of the trace is not 0	Assessment The volume limit is along a subsequence, and uniqueness has not been shown. The uniqueness of the vacuum of the theory before decomposition has not been shown. Assessment What the committee compared with independently written OS axioms is only the scalar hierarchy of the trace in the chosen phase (§ 10.1)	`Hierarchy.Reconstr` (line 117), `Properties.reconst` `nonfree` (lines 177- Assessment § 5, § (
2 The local fields of Yang–Mills	Assessment It takes as labels the space V_d of gauge-invariant polynomials in the curvature of weight at most d, and takes as the space of fields $F_d = V_d/N_d$, the quotient by the part N_d whose correlations at separated points all vanish. Assessment F_d is linearly isomorphic to the space of polynomials of the free Maxwell theory	Source In the public statement the type of labels `Token` is an abstract type, and its meaning as polynomials in the curvature is not written (it is on the side of `C.Makes` in the construction theorem). Source An example among the committee's checks writes that a linear isomorphism is not an agreement of correlations (lines 84–86 of the same file). Source An example among the committee's checks writes that the labels are polynomials at a point, and that completeness is not shown for non-local observables such as line defects (`submissions/openai-20261028/docs/examples/OperatorContent-20261028.lean`, lines 35–38)	`Hierarchy.Field` (l `Token` in `PublicWilson.lean` 44). Assessment § 3 § 9.4

Requirement	What the construction gives	What is not reached	Location, assessme
3 Short-distance behaviour	Source The bare coupling approaches a positive slope in $\beta_{N,p}/\log(1/aN)$ (uniformly in the volume). Assessment The stress tensor is a point on a one-dimensional space of conserved symmetric tensors, normalised by the Ward identity. Assessment The short-distance law (2) for the two-point function of the trace. Assessment An operator product expansion with error bounds. Assessment Along one sequence of scales, the correlations of the stress tensor approach those of the free Maxwell field	Assessment Agreement with perturbation theory at all orders has not been shown. Assessment The coefficients of the operator product expansion are not compared with perturbative coefficients. Source In the public statement, the trace field is an arbitrary element of F_4, and it is not written that it is the trace of the stress tensor. Source There are three declarations of a form close to the short-distance effective coupling (assessment § 7.4), and none survives into the conclusions of the three targets: `Running.ratio` is outside the dependencies, `wilson_link` is an intermediate step inside the dependencies (its wording is close to Theorem 7.2 Inferred), and the headline `Asymptotics.comparison` is the coupling at a step with a cutoff, a different quantity from g_W (6-00 § 05)	`WilsonRegularity.r` (lines 13–14), `Trace` (lines 168–175), `OperatorProductExp` Assessment § 7
4 The mass gap	—	Assessment Not claimed. Source What there is are polynomial bounds uniform in each phase (the spectral weight at low energy is at most Mu^2, the heat bound at most $M\tau^{-2}$), not exponential decay	`light_bound` in `YM/EveryPhase/Ligh` (line 221), `inverse_` in `YM/EveryPhase/Deca` (line 173). Assessm § 8

The short-distance law in the third row of Table 1 has the following form for the two-point function of the centred trace Θ in the chosen phase ξ_* Source (`ChallengeSpec.lean`, lines 173–174; formula (16) of assessment § 7.2).

$$\lim_{r\downarrow 0}\; r^{8}\,\log^{2}(1/r)\; S^{\xi_*}_{\Theta,\Theta}\big(-\tfrac{r}{2}e0,\ \tfrac{r}{2}e0\big) \;=\; \frac{3\,\dim_{\mathbb{R}}\mathfrak{g}}{\pi^{4}}. \qquad (2)$$

r^{-8} is the naive power of the two-point function of a field of dimension 4. The $\log^{-2}$ is a trace of the fact that the trace has a coefficient that becomes small through the running of the coupling Inferred . Because this logarithm is a form that does not arise from polynomials in free fields of finitely many masses, "not free" is concluded (13-02).

The polynomial bounds in the fourth row are of a general form that follows from local commutativity and the forward light cone Inferred . They are compatible with a massless theory too, so they do not replace (1).

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04
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The committee's standard of "existence", and its reservations

The committee writes for itself at what strength it accepted "existence".

- **Subsequences.** The infinite-volume limit is taken only along a subsequence obtained by compactness (Arzelà–Ascoli) Assessment . Assessment § 5 writes that the Jaffe–Witten problem statement regards "a subsequential limit obtained by compactness as insufficient without a separate argument showing properties of the limit", and then states that the reconstruction and the theorems on local fields give information beyond compactness Assessment .
- **The vacuum.** The assessment writes that the standard of existence adopted here "does not require a precise characterisation of the vacuum state" Assessment .
- **Missing uniqueness.** The committee writes that the fact that uniqueness of the infinite-volume limit and of the vacuum has not been shown does not contradict the general conjecture that they are unique Assessment . A conjecture is a conjecture, not something shown.
- **Degree of understanding.** The committee does not claim yet to understand the proof fully. What was used for the assessment is the Lean source; the manuscript of the paper proof was not used Assessment .

In addition, the public theorem (`JointExistence`) originally did not include part of the connection with the lattice in its statement. Assessment § 9 writes that the committee was informed of this by OpenAI, and that the committee's verdict rests on a stronger combined target Assessment . This difference is read in 6-02 § 05.

In sum, what the opening verdict says is "proved" is the construction of requirement 1, at a strength that includes subsequences and the choice of a phase. Requirements 2 and 3 are present in partial form: labels and quotients of local fields, and short-distance laws at the level of the leading terms. The individual short-distance results are stated as theorems of the submission, and the assessment calls them together evidence "that it is Yang–Mills". Requirement 4 is, before any question of reaching it, outside what is claimed.

05

What this book's investigation had seen

In Volume 3, this book has statements corresponding to some of the four requirements, **on a single lattice with the lattice spacing held fixed**. It shows nothing about the continuum theory. Table 2 places them in the same rows as Table 1.

Table 2 The four requirements, and where the results of Volume 3 of this book lie (all statements on this

book's side are at fixed lattice spacing)

Requirement	The published construction	Volume 3 of this book
1 Axioms	A Wightman theory for each phase, in the continuum limit	On the lattice inside the strong-coupling window: the infinite-volume limit without subsequences, reflection positivity, reconstruction (20-03, 20-04). No continuum limit
2 Local fields	The space of labels and the quotient Fd	None. This book's observables are plaquette averages, Wilson loops and ratios of twisted partition functions
3 Short distances	Running of the coupling, stress tensor, trace, operator product expansion	Components at the one-loop level, and a theorem at one ultraviolet scale (latter half of Part 15). No control over all scales
4 The mass gap	Not claimed	Only on the lattice inside the strong-coupling window, the gap of the transfer operator (20-04). The vortex floor (21-03) is not a statement of the mass gap

Of what this book has listed as "statements to be shown" (19-07), the part corresponding to existence on the ultraviolet side is answered by the published construction at the level of the committee's assessment. The side of uniform decay and the mass gap remains open on both sides (6-04).

06

What has not been read

- The four of § 01 are the assessment's summary. This chapter has not matched them one by one against the sentences of the original Jaffe–Witten text (for a reading of the original, 0-03).
- We have not found the Lean statement corresponding to "the theory of perturbative renormalisation" in requirement 3. Nor have we checked the location of the second coupling of assessment § 7.4 (the one measured by the tensor–scalar correlation). We infer that it is near `YM/Pert/Position/Four.lean` Inferred .
- We have read the fields of the reconstruction statement (`Hierarchy.Reconstructed`) as a list, but have not matched each field one by one against the axioms of 4-09. The committee's independent comparison (assessment § 10) concerns the scalar hierarchy of the trace Assessment .
- We have not read the proofs of the polynomial bounds (`light_bound`, `inverse_square`). That they come from local commutativity is inferred from the names of the lemmas in the proofs Inferred .

What this chapter can claim

Said	Not said
Assessment The committee's assessment is that requirement 1 (the construction of a theory satisfying the Wightman axioms) is proved in Lean, in a form that includes subsequences and the choice of a phase	Uniqueness of the infinite-volume limit, uniqueness of the vacuum of the theory before decomposition
Assessment Requirements 2 and 3 are present as labels and quotients of local fields and as short-distance laws at the level of the leading terms. The individual short-distance results are stated as theorems of the submission, and the assessment calls them together substantial evidence "that it is Yang–Mills"	Agreement with perturbation theory at all orders, a complete identification of the local fields
Source Statements of the form in Table 1 are at the locations in Table 1	The correctness of the proofs (not read)
Assessment Requirement 4 (the mass gap) is not claimed	—
This book's results in Table 2 are statements at fixed lattice spacing	This book shows nothing about the continuum limit, weak coupling or the substance of the mass gap

Sources and reproduction

Item	Type	Source, tool
The four requirements	Assessment	Assessment § 1. The original is the Jaffe–Witten problem statement (reference [JW] of the assessment). For how to read it, 0-03
The verdict, and what the construction gives	Assessment	Assessment opening, § 2
The fields of the public statement	Source	OpenAI-YM-20261028: `ChallengeSpec.lean`, lines 12–48 and 117–181; `PublicWilson.lean`, lines 35–46
Polynomial bounds	Source	`YM/EveryPhase/Light.lean`, line 221; `YM/EveryPhase/Decay.lean`, line 173
Subsequences, the vacuum, degree of understanding, the manuscript	Assessment	Assessment opening, § 5, § 9
The three locations of the short-distance effective coupling	Source	`YM/PertExtension/Running.lean:145` (outside), `YM/TraceExtension/Atlas/Sink.lean:50` (used), `YM/Pert/Asymptotics/RawAxis.lean:36` (headline). `verification/declaration-usage.tsv`
This book's results	—	The source tables of each chapter of Parts 20 and 21

Terms used in this chapter — mass gap, Wightman axioms, OS axioms, phase decomposition, field labels, trace anomaly, operator product expansion, subsequential limit. Their meanings are in A-02 Glossary.

Next chapter to read: 6-02 Reading the statements — reading in full the public theorem to which the "Location" column of Table 1 belongs, and setting out how it differs from the construction theorem and the committee's combined target.

Reading the statements — the public theorem, the construction theorem, the committee's combined target

> This chapter is existing knowledge (a reading of the published construction). § 08 is this book's own study.

What this chapter shows — The full text of the public theorem and its logical form. What sits inside the existential quantifiers (what the reader chooses, what the prover chooses, and the conditions the statement imposes). What the construction theorem adds to the public theorem. What the combined target the committee wrote itself adds. How the three scopes differ (Table 3). What the verification record checked and what it did not.

Prerequisite chapters — 6-00 Entrance (§ 05, distinguishing the targets), 6-01. For how to read Lean, see A-04.

Said first — What this chapter reads is the form of the statements, not the content of the proofs. This book has not repeated the verification on its own machines. All results of the verification are taken from the assessment report and the copies of the logs Assessment .

The order of this chapter

1. The public theorem — the full text and its logical form
2. What sits inside the existential quantifiers — who chooses what (Table 1)
3. The four conclusions of the public theorem — wilson, reconstruction, ope, nonfree
4. The construction theorem — the penalty coefficient, the family certificate, refining the order and the bundle of conclusions
5. The committee's combined target — checks on one family and one witness (Table 2)
6. How the three scopes differ — item by item (Table 3)
7. What the verification record checked, and what it did not — rebuild, kernel, axioms, Comparator (Table 4)

8. What this book's study had seen — a comparison with the form of this book's statements
9. What has not been read

Labels: Source = we read the source of the published construction / Inferred = this book's inference / Assessment = from the committee's assessment report. The other labels (Proved here, Cited, Reading, Lean and so on) mean the same as in the rest of the book (see "Labels" in 6-00). Unless stated otherwise, paths are relative to the root of the repository `OpenAI-YM-20261028`.

01

The public theorem

The statement in the form of the public question is at lines 186–188 of `ChallengeSpec.lean` Source. Because the form matters, here are the three lines verbatim.

```
def JointExistence : Prop := ∀ g : GaugeGroup, g.IsSimple → ∃ C : WilsonFamily g,
  WilsonRegularity C ∧ ∀ p : ℕ → ℕ, StrictMono p → ∃ q : ℕ → ℕ, StrictMono q ∧
    ∃ T : Hierarchy C, T.Properties (p ∘ q)
```

In mathematical language this reads as follows.

For every closed subgroup of matrices $G \subset U(m)$ whose real Lie algebra $\mathfrak{g}$ is simple, there is a Wilson family C such that the following holds. C satisfies the four conditions of regularity, and however a strictly increasing sequence p of volume indices is given, there are a refinement q of it (strictly increasing) and a package T of continuum theory such that T has four properties along the refined sequence $p \circ q$.

The proof is `jointExistence` in `Solution.lean` (4 lines); it calls the construction theorem with penalty coefficient 1 and merely extracts part of its conclusion Source.

"Simple" is defined by `IsSimple` in `PublicTheory.lean` (line 39): the Lie algebra is not commutative (there is a pair of elements whose commutator is not 0), and it has no ideals other than 0 and the whole Source. This is the only hypothesis of the public theorem Source.

02

What sits inside the existential quantifiers

The most important thing in reading the public theorem is that the content of each $\exists$ is **chosen by the prover**. What the statement imposes is only the conditions the chosen objects must satisfy. Table 1 separates the reader's side ($\forall$) from the prover's side ($\exists$).

Table 1 What sits inside the quantifiers of the public theorem (located in `PublicWilson.lean` and `ChallengeSpec.lean`)

Object	Chosen by	Content	What the statement imposes
Group G	Reader ($\forall$)	Closed subgroup of matrices	Hypothesis: the Lie algebra is simple
Wilson family C	Prover ($\exists$)	Side of the volume L_p, denominator D_N (spacing $a_N = 1/D_N$), bare coupling $\beta_{N,p}$, measure on the group, type of labels, lattice insertion for each label (`WilsonFamily`, lines 35–46)	L_p and D_N strictly increasing. The measure is a probability measure and is Haar measure. The four conditions of regularity (below)
Sequence of volumes p	Reader ($\forall$)	Strictly increasing sequence	—
Refinement q	Prover ($\exists$)	Strictly increasing sequence	—
Package of continuum theory T	Prover ($\exists$)	Hilbert space and Wightman theory, spaces of fields by degree, map from labels to fields, space of phases and a measure on it, the chosen phase, correlations, analytic continuation, coefficients of the operator product expansion, the centred-trace field (`Hierarchy`, lines 24–48)	Four properties `Properties` (lines 177–181)

The four conditions of regularity `WilsonRegularity` (lines 12–22) are as follows `Source` .

- **Running**: there is a positive number s (the slope) with $\beta_{N,p}/\log(1/a_N) \to s$. The convergence is uniform in the volume p.
- **Continuity**: each insertion is continuous in the link configuration.
- **Gauge invariance**: each insertion is unchanged by lattice gauge transformations.
- **Locality**: for each label there is a finite stencil (a finite set of displacements in lattice units), and the insertion depends only on the links inside that stencil. The stencil does not depend on the spacing, the volume or the position.

Three things can be read from Table 1.

- The statement asks only that a positive slope s exists; no value is in it Source . The value the construction satisfies is $s = 11C_{\mathfrak{g}}/(24\pi^2)$ (assessment report, Appendix A) Assessment .
- The type of labels is an abstract type, and which polynomial in the curvature a label represents is not in the statement Source . The centred-trace field is an arbitrary element of the space of fields of degree 4 (line 48) Source . The only thing that constrains the trace field is the condition in the fourth property of § 03.
- The sequence of spacings a_N is also chosen by the family Source . So universality in the sense "it converges for every sequence of spacings" is outside the statement (assessment report § 4.3) Assessment .

03

The four conclusions of the public theorem

We read the four fields of `Properties` for their meaning Source .

1. **wilson** (line 178): the link to the lattice. The lattice correlation $C_{N,p}(v;t)$, smeared with a test function t on configurations of mutually distinct points, has a limit as $N \to \infty$ for each volume p in which it fits, and that limit approaches $\int t\, S^{\mathrm{mix}}$ along the refined sequence of volumes (`Converges` in `PublicWilson.lean`, lines 85–90). The continuum limit runs along the whole sequence of spacings chosen by the family. It is the volume limit that is taken along a subsequence.
2. **reconstruction** (line 179): the Wightman axioms, faithfulness of the fields, the phase measure being a probability measure with full support, permutation symmetry and smoothness of the correlations, continuity in the phase, the average over phases being equal to the mixed correlation, analytic continuation and boundary values (`Hierarchy.Reconstructed` , lines 117–143). The axioms, including uniqueness of the vacuum, and the accompanying fields (faithfulness, analytic continuation, boundary values) are stated for the theory of the chosen phase. The Wightman axioms in every phase are on the side of the construction theorem (§ 04).
3. **ope** (line 180): the coefficients of the operator product expansion, and a power-law upper bound on the remainder after truncating by degree. The bound on the remainder is uniform in the state (13-04).
4. **nonfree** (line 181): the two-point function of the centred-trace field satisfies, in the chosen phase, the short-distance law of equation (2) in 6-01, and agrees near 0 with no finite sum of Wick forms of finitely many mass shells (lines 168–175). One of these fields, `amplitudePositive` , is an inequality between constants that does not involve the theory T.

All correlations are defined only where the points are mutually distinct Source . Where points coincide is outside the public theorem (6-04).

04

The construction theorem

The construction theorem is `constructed` in `Construction.lean` (line 15) Source . Its form is as follows.

For each simple G and each positive **penalty coefficient** c_{pen} (in the source's wording `kernel.penalty`: the coefficient of the weight in the one-step probability kernel that pulls the coarse links towards the predictor, 9-01), there is a family C such that the following hold. The penalty coefficient of the kernel equals c_{pen}. The family certificate `C.Makes` holds. The four conditions of regularity hold. Moreover, however a pair of subsequences of spacings and volumes (the order `Extraction.Order`) is given, there are a refinement of it, together with a table of correlations, a final witness, a bundle of reconstructions in every phase (`EveryPhase.Almanac`), a reader of the operator product expansion, contact apparatus and a sequence of scales, such that a dozen or so conclusions hold at once Source .

The main things added to the public theorem are these four.

- **The family certificate** `C.Makes` (`YM/Source/Construction.lean` line 44): the renormalisation relation at every step (`path.Selected`), soundness of the local sections, the label stencils fitting in a box, and the insertion coefficients being exactly the label coefficients when read at the reference step (`calibrated`) Source . The meaning of a label as a polynomial in the curvature, and its renormalisation, are fixed here (assessment report § 9.4) Assessment .
- **The strong form of the ultraviolet limit** `FullVolume.PositionsUV C Extraction.Sequence.full`: for each volume there is a kernel, smooth where points do not coincide, and the correlations approach it locally uniformly, with differences, along the whole sequence of spacings Source . The wilson field of the public theorem is only convergence of smeared values; this form is only in the conclusion of the construction theorem Source .
- **Identification of the trace**: the centred-trace field is the trace of the stress tensor minus its vacuum expectation value (`HierarchyBridge.lean` line 42, `identified` at lines 87–91 of `YM/Nonperturbative/HeadlineSpec.lean`) Source .
- **Every phase**: the Wightman axioms not only in the chosen phase but in every phase Source .
- **Comparison with the free Maxwell field**: the correlations of the mixed theory (the average over phases) approach those of the free Maxwell field along one sequence of

scales (`Pert.FirstExtension`). The source's docstring says "only the averaged correlations" (`YM/Pert/FirstTarget.lean` line 4) Source .

The proof of the construction theorem descends the chain of main theorems `Nonperturbative.existence` (`YM/Nonperturbative/Headlines.lean` line 19) → `continuum_with_position_tests` → the choice of one run `constructionRun` (`YM/Independent/ConstructionRun.lean` line 14) Source . It is organised so that a run is chosen only once for each penalty coefficient, and every subsequent object is built on that run Source . The rest of the chain is read in 6-03.

05

The committee's combined target

According to § 9 of the assessment report, after submission OpenAI informed the committee that the original public target `jointExistence` had dropped part of the link to the lattice construction that the proof sets up Assessment . The committee wrote a target of its own that keeps that link, and based its judgement on it Assessment . That target is `YMReview.Acceptance.combined_acceptance` (line 111 of `submissions/openai-20261028/checks/CombinedAcceptance.lean` in the repository `YM`; the type of the statement is `CombinedAcceptance` at line 172 of `AcceptanceSpec.lean`) Source .

Its form is: "for each penalty coefficient one family is fixed first, and for every order there are one refinement, one table and one final witness, and all the check fields hold on those **same** objects" Source . The proof calls the construction theorem and transfers its conclusions to the committee's fields Source . Table 2 lists the fields.

Table 2 The fields of the committee's combined target (`AcceptanceSpec.lean` lines 34–170)

Field	Content (field names and types read at the source; meaning per the section on the right)	Asses
Lattice and scales	The family certificate and regularity. The finite-lattice measure agrees with the Wilson measure of an independently written lattice gauge theory (coupling conversion $\beta_{\mathrm{lgt}} = \beta/m$). The order on the whole sequence of spacings. The public limit. The strong ultraviolet form. Reference spacing $a_0 = 1$. The step-0 coupling has the same positive value for all N, p. Normalisation of the read-out of insertions	§ 11 (
Phases and OS	The phase measure is a probability measure with full support. The average over phases. The centred-trace hierarchy of the chosen phase satisfies independently written OS axioms and a growth condition (E0′). That trace is the same as the trace of the construction	§ 10 (
Translation Ward identity	In a form with fixed signs and coefficients, for every phase and every field. Integrability of the integrand, and independence of the choice of cut-off function	§ 11 (
Operator content	Every invariant polynomial is among the labels. A linear isomorphism between the space of fields of each degree and the space of free Maxwell shells (also in each phase). Density of words, and faithfulness	§ 3, §

Field	Content (field names and types read at the source; meaning per the section on the right)	Asses
Leading term of the trace	The short-distance law from every direction, for fields of the same chosen phase	§ 7.2 (
Positive lattice probe	For a fixed test function, the double limit has a nonzero value	§ 11 (
Source conclusions	The bundle of conclusions of the construction theorem, kept as is	§ 9.3

The docstring of `AcceptanceSpec.lean` itself says that bundling the source conclusions is not a check of the independent meaning of each definition[Source] . The matching of the operator-product-expansion statement (`YMReview.OPE.constructed`) is an endpoint separate from the combined target (assessment report § 11)[Assessment] .

06

How the three scopes differ

Table 3 The public theorem, the construction theorem, the committee's combined target (○ in the statement, — not. All[Source])

Item	Public theorem	Construction theorem	Combined target
Hypothesis	Lie algebra simple	Simple, penalty coefficient $c_{\mathrm{pen}} > 0$	Same
Lattice measure is the Wilson measure	○ (definition of the family's measure)	○	○ also agrees with an independent definition
Labels represent polynomials in the curvature; renormalisation coefficients	— (labels are abstract)	○ (family certificate)	○
Continuum limit (smeared values)	○ whole sequence of the family's spacings	○	○
Continuum limit (smooth kernel, with differences)	—	○	○
Volume limit	Subsequence	Subsequence	Subsequence
A unit common to all spacings and volumes ($a_0 = 1$, common step-0 coupling)	—	Inside the family certificate	○ as a field
Wightman axioms	Chosen phase	Every phase	Every phase (source conclusion)

Item	Public theorem	Construction theorem	Combined target
Matching against independently written OS axioms	—	—	○ only the trace scalar of the chosen phase
Trace field is the trace of the stress tensor	—	○	○
Translation Ward identity	—	○ (as the stress law)	○ in a form with fixed coefficients
Free Maxwell field at short distance (correlations of the mixed theory, one sequence of scales)	—	○	○ (source conclusion)
Short-distance effective coupling (the theorem of assessment report § 7.4)	—	— (not among the conclusions; a lemma with similar wording is an intermediate step inside the dependencies)	—
Mass gap	—	—	—

The last two rows of Table 3 matter. The body of the assessment report lists the short-distance effective coupling as Theorem 7.2, but does not name where it is in Lean Assessment . The source has three declarations of a similar form Source .

- `YM.PertExtension.Running.ratio` (`YM/PertExtension/Running.lean:145`): a ratio form ($\mu \to \infty$, $a_m \leq 1/\mu$). Outside the dependencies (outside) Source .
- `YM.Trace.Pilot.wilson_link` (`YM/TraceExtension/Atlas/Sink.lean:50`): inside the dependencies (used; the first headline is `continuum_with_position_tests`, the direct user is `crest_ready`), an intermediate step of the trace analysis Source . It has `PostT` among its hypotheses, and its conclusion enters an intermediate certificate and then drops out of the conclusions of all three targets Source . This is the one whose wording is close to the statement of Theorem 7.2 ($\log(1/r)\, g_W(r)^2 \to 1/(2b_0)$) Inferred .
- `YM.Pert.Asymptotics.comparison` (`YM/Pert/Asymptotics/RawAxis.lean:36`): itself a headline theorem (headline), the running of the step-i coupling with cut-off $\beta(N_i, k_i, i)^{-1/2}$. This is a different quantity from g_W Source .

Volume 2 cites the short-distance effective coupling as "there are declarations of a similar form, but none survives in the conclusions of the public theorem, the construction theorem or the committee's combined target" (10-06, 13-03).

> 07

What the verification record checked, and what it did not

§ 9 of the assessment report separates formal verification (the proof derives the statement from the definitions and the permitted axioms) from validity (the formal statement correctly represents the intended problem)[Assessment] . Table 4 lists, check by check, what was checked and what was not.

Table 4 The verification record (all [Assessment] ; the lines of the log copies were checked at the [Source])

Check	What was checked	What was not checked
Rebuild (§ 9.1)	With Lean 4.32.0, 9,470 modules rebuilt with no pre-existing artefacts, 0 errors (about 184 minutes). 9,477 source files agree with the submission's manifest	2,453 warnings remain (mostly unused variables and the like)
Inventory (§ 9.1)	Among the 250,319 declarations loaded, no custom axioms, no direct references to `sorryAx`, no `unsafe` marks	—
Kernel recheck (§ 9.1)	By the submitter's checker `VerifyJoint.lean`, separate from Comparator. The 100,168 declarations on which the 17 headline theorems depend were rechecked in a fresh kernel environment initialised only with the external libraries. 14,364 external declarations at the boundary. The only axioms are `propext`, `Classical.choice` and `Quot.sound`	Declarations not reachable from the 17 headlines (about 150,000, the rest of the inventory) were not rechecked
Comparator (§ 9.2)	Against fixed reference statements and definitions: agreement of statements and definitions, axioms, kernel recheck. The public theorem (10-30), the combined target (11-03) and the operator-product-expansion matching (11-05) passed. Four controls (a correct proof passes; versions with altered statements or definitions, and forbidden axioms, are rejected)	Run on macOS, without a check that isolates potentially malicious build code. No kernel from another implementation was used
Human reading (§ 9.3)	The definitions in the appendix glossary were read against the problem statement and the conjecture	Not exhaustive, the assessment report says
Matching against independent axioms (§ 10, § 11)	The trace hierarchy of the chosen phase satisfies OS axioms written independently beforehand. The finite Wilson measure agrees with an independent definition. The Ward identity, the common unit, the trace law	The analytic estimates themselves rest on the submitted proof. The matching concerns the hierarchy of the trace scalar, not every field. Test functions flat on the diagonal do not see contact terms

The log copy `verification/joint-check.log` records the type match of the public theorem (`TYPE_MATCH YangMills.jointExistence`) and the numbers of the recheck (project 100,168, external 14,364, endpoints 17)[Source] . The assessment report notes as

a concern about code quality that, compared with the total number of declarations, few enter the dependencies of the main theorem Assessment .

Among the things not checked, two bear on this book's reading. One is that the kernel's acceptance does not decide that the definitions express the intent (§ 9.3) Assessment . The other is that the committee's OS matching is limited to the hierarchy of the trace scalar (§ 10.1) Assessment .

08

What this book's study had seen

This book's Lean consists of two kinds of statement (19-00): theorems about concrete objects defined on the lattice, and implications with hypotheses placed in fields (chain theorems). The closest forms in the published construction are these three Inferred .

- An implication that takes estimates as hypotheses and concludes a limit (`positioned` in `YM/FullVolume/Profiles.lean`, line 42) Source .
- A certificate bundling the family certificate, the order, pointwise limits and the table's axioms into one (`Seed` in `YM/Spec/Closed.lean`, line 20) Source . Reading this as "an assembly that transfers properties to the limiting table under conditions" is this book's inference Inferred .
- An arrangement in which the requested propositions are placed outside as types, closed by proofs elsewhere and passed to the theorem above Inferred .

The difference is the endpoint. The published construction closes every hypothesis at the end and arrives at a statement whose only hypothesis is "simple" Assessment . This book's chain theorems stop with premises such as uniform estimates across steps still in place (28-04). Another difference is that the family is chosen inside the existential quantifier. This book's chain took the sequence of bare couplings as given from outside, not as determined by matching (14-03).

09

What has not been read

- Of the dozen or so conclusions of the construction theorem, we have not read the contents of `Independent.Core`, `Pert.Position.FourMeters` or the fields of the contact apparatus.
- We have not read the bridge in `HierarchyBridge.lean` (the transfer to the public types), except the lines on the trace field.
- We have not read the proofs in the committee's check files (`CompleteOS.lean`, `CompletenessAudit.lean`, `LGTWilsonBridge.lean` and others). We read only the types

of the statements (`AcceptanceSpec.lean`).

- We have not computed the hash values in the log copies ourselves to compare them.
- Among the 17 headline theorems, we have not read what those outside the line of the public theorem and the construction theorem (for example `YM.SharpGram.habitat`) say.

What this chapter can say

Can say	Cannot say
Source The only hypothesis of the public theorem is "the Lie algebra is simple"; the family, the refinement and the package of theory sit inside the existential quantifiers	That the proof is correct (this is at the level of the assessment)
Source In the public theorem, the continuum limit is convergence of smeared values along the whole sequence of the family's spacings, and the volume limit is along a subsequence	Universality with respect to the choice of the sequence of spacings; uniqueness of the volume limit
Source The meaning of labels, the identification of the trace and the strong ultraviolet form are on the side of the construction theorem	Reading them from the public theorem alone
Assessment The judgement rests on the committee's combined target	That the checks of the combined target are a check of the independent meaning of every definition
Source There are three declarations of a form close to the short-distance effective coupling, and none survives in the conclusions of the three targets	Which of them is where Theorem 7.2 of the assessment report lives (the report gives no name)

Sources and reproduction

Item	Type	Source / tool
Public theorem	Source	OpenAI-YM-20261028: `ChallengeSpec.lean` lines 12–48 and 117–188, `PublicWilson.lean` lines 35–90, `PublicTheory.lean` line 39, `Solution.lean` 4 lines
Construction theorem	Source	`Construction.lean` lines 15–40, `YM/Source/Construction.lean` lines 44–65, `HierarchyBridge.lean` lines 27–42, `YM/Nonperturbative/HeadlineSpec.lean` lines 84–91, `YM/Nonperturbative/Headlines.lean` lines 19–22, `YM/Independent/ConstructionRun.lean` line 14
Combined target	Source	yang-mills-group/YM (revision 55ba9fc): `submissions/openai-20261028/checks/AcceptanceSpec.lean` lines 34–176, `CombinedAcceptance.lean` lines 111–121

Item	Type	Source / tool
Why the combined target was written	Assessment	Assessment report § 9 (opening), § 9.3, § 9.4
Verification record	Assessment	Assessment report § 9.1, § 9.2, § 10, § 11, § 12.2. Log copies `verification/joint-check.log`, `build.log`
Inside and outside the dependencies	Source	`verification/declaration-usage.tsv`, `verification/Usage.lean`, `verification/joint-headlines.txt`
Short-distance effective coupling	Source	`YM/PertExtension/Running.lean:145` (outside), `YM/TraceExtension/Atlas/Sink.lean:50` (used), `YM/Pert/Asymptotics/RawAxis.lean:36` (headline). Assessment report § 7.4
Only the averaged correlations	Source	`YM/Pert/FirstTarget.lean` lines 1–5. Assessment report § 7.4

Words used in this chapter — existential quantifier, Wilson family, four conditions of regularity, label, penalty coefficient, family certificate, order and refinement, combined target, Comparator, kernel recheck. Their meanings are in the A-02 Glossary.

Next chapter to read: 6-03 The skeleton of the proof — descending the chain of the construction theorem, we see the ten stages from the calendar to the evidence in one figure and one table.

The skeleton of the proof (one picture)

> This chapter is existing knowledge (a reading of the published construction). § 06 is a guide, and § 07 is this book's own study.

What this chapter shows — In what order the published construction builds what, from the Wilson measure on the lattice to the Wightman theory and the "evidence that it is Yang–Mills". What each of the ten stages defines and what it shows (with equations). The interfaces that connect the stages. Where the main declarations of each stage are, and the chapters of Volume 2 that treat them. How the roughly 990,000 lines of code are distributed over the stages.

Prerequisite chapters — 6-02 Reading the statements (the construction theorem, and the choice of one run). For the idea of renormalisation steps, see 7-05 The sequence of renormalisation group steps; for reflection positivity and the transfer matrix, 5-03.

Said first — This chapter is an overview. The statements of each stage are readings of their form; the content of the proofs is treated in the chapters of each part, only as far as we could read it. The division into stages is this book's arrangement for readability and does not match how the code is laid out. The distribution of line counts is a rough count by unit of location Inferred .

The order of this chapter

1. One picture — ten stages, and one run (Figure 1)
2. Stages 1–5: renormalisation on the lattice — calendar, one step, representation and induction, shooting, calibration
3. Stages 6–10: limits and reconstruction — ultraviolet limit, infinite volume, Euclidean invariance and phases, reconstruction, evidence
4. What connects the stages — the push-forward identity, continuous curves, the root seminorm, the endpoint
5. The size of the code and its distribution over layers — a count by unit of location (Table 2)
6. A guide to the chapters of Volume 2 — chapters by stage (the right-hand column of Table 1)
7. What this book's study had seen — a comparison with this book's set-up

8. What has not been read

Labels: Source = we read the source of the published construction / Inferred = this book's inference / Assessment = from the committee's assessment report. The other labels (Proved here, Cited, Reading, Lean and so on) mean the same as in the rest of the book (see "Labels" in 6-00). Unless stated otherwise, paths are relative to the root of the repository `OpenAI-YM-20261028`.

01

One picture

For each penalty coefficient $c_{\mathrm{pen}} > 0$, the construction theorem chooses, only once, a "run" that satisfies certain conditions (`constructionRun` in `YM/Independent/ConstructionRun.lean`, line 14) Source . Every later object is built on this one run. Objects whose existence is shown separately might not be compatible when combined afterwards, so the design gathers the existence proof into a single line Inferred . Figure 1 shows the ten stages on that run.

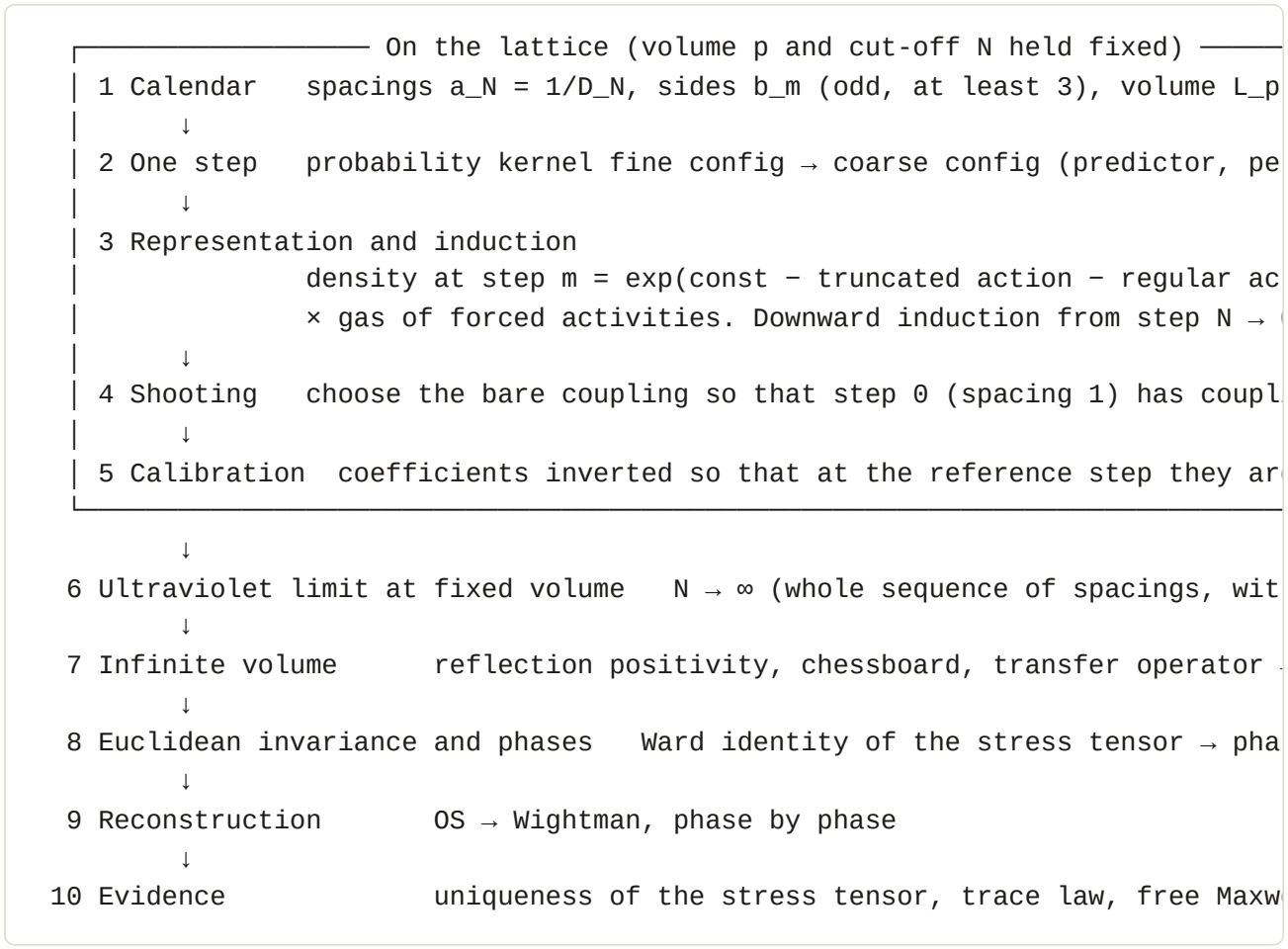

Figure 1 The skeleton of the proof. Stages 1–5 are renormalisation on the lattice; all the steps are built with the volume and the cut-off held fixed. Stage 6 removes the cut-off, and stage 7 removes the volume.

Table 1 summarises, for each stage, its content, its main declarations and the chapters that treat it. The label on the content means that we read the form of the statement in that row at the source.

Table 1 The ten stages (all content `Source` ; correctness of the proofs is at the level of the assessment)

Stage	What it defines and what it shows	Main declarations (location)	Chapters
1 Calendar	Sides b_m (odd, at least 3), volume sides L_p (multiples of 4, strictly increasing), $D_N = \prod_{k<N} b_k$. Constants for each step and the reference coupling $\bar{\beta}$	`Schedule` (`YM/Lattice/Scale.lean` line 28), `Calendar` (`YM/RG/History.lean` line 64)	8-02, 10-05, 11-08
2 One step	A probability kernel from fine configurations to coarse configurations	`StepData.density`, `push` (`YM/RG/Block.lean` lines 71, 77)	9-01 to 9-05

Stage	What it defines and what it shows	Main declarations (location)	Chapters
3 Representation and induction	The representation of the density at each step, the induction hypothesis, the one-step relation, the one-step theorem	`Rep`, `History.Conv`, `Transfer`, `Selected` (`YM/RG/History.lean` lines 165, 213, 223, 234), `Directed.Program` (`YM/PCalc/Band.lean` line 65)	9-06, 9-07, 11-00 to 11-09
4 Shooting	Choosing the bare coupling so that the step-0 coupling matches $\bar{\beta}$	`Arc.bought` (`YM/FullVolume/Arc.lean` line 95); on the path of the main theorem, `rodB` (`YM/FullVolume/Stage/Rods.lean` line 26)	10-04
5 Calibration	Fixing the insertion coefficients by inverting the renormalisation transport	`calibrated` in `Family.Makes` (`YM/Source/Construction.lean` lines 44–65)	8-04, 11-10
6 Ultraviolet limit	For each volume, convergence along the whole sequence of spacings, with kernels and differences	`Campaign.realUV` (`YM/FullVolume/Fields/Filled.lean` line 45), `positioned` (`YM/FullVolume/Profiles.lean` line 42)	11-10
7 Infinite volume	Reflection positivity, the chessboard inequality, input from the ultraviolet, equicontinuity, subsequential limits	`reflected_positive` (`YM/Slice/Reflection.lean` line 98), `tiles`, `adds` (`YM/Cells/Chess.lean` lines 80, 108), `Wcmain` (`YM/Extraction/Center.lean` line 38), `first_surface` (`YM/Extraction/Lipschitz.lean` line 166)	12-01 to 12-05
8 Euclidean invariance and phases	The stress tensor and the Ward identity, vanishing of the rotational flux, phase decomposition	`YM/Euclid/Stress.lean`, `OS.Selection` (`YM/Phase/Mixture.lean` line 127)	12-06, 12-07
9 Reconstruction	OS reconstruction and the Wightman axioms in each phase	`Hierarchy.Reconstructed` (`ChallengeSpec.lean` line 117), `EveryPhase.every` (`YM/EveryPhase/Vacuum.lean` line 146)	12-08
10 Evidence	Uniqueness of the stress tensor, the short-distance law of the trace, comparison with the free Maxwell field, the operator product expansion	`YM.Pert.one_line` (`YM/Pert/Weak.lean` line 68), `ChallengeSpec.lean` lines 168–175, `YM/Pert/FirstTarget.lean`	13-01 to 13-04

```
02
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Stages 1–5: renormalisation on the lattice

Stage 1 Calendar

The lattice spacing is $a_N = 1/D_N$, with $D_N = \prod_{k<N} b_k$ Source . The sides b_m are odd numbers at least 3 and grow with the step, but $\log b_m / \log D_m \to 0$ (`Schedule.Gentle`) Source . The volume is a torus of side L_p in physical units, which on the lattice has $L_p D_N$ points per side Source . The spacing at step 0 is 1, and this becomes the common unit of length Assessment .

The calendar fixes reference values for each step Source . With $C_{\mathfrak{g}}$ the Casimir constant of the adjoint representation,

$$\hat{\beta}_m = \bar{\beta} + \frac{11 C_{\mathfrak{g}}}{24\pi^2} \log \frac{1}{a_m}, \qquad \hat{g}_m = \hat{\beta}_m^{-1/2}, \qquad \mathrm{monitor}_m = \exp\big((\log\log \hat{\beta}_m)^2\big). \tag{1}$$

The small-field threshold is $\hat{g}_m \cdot \mathrm{monitor}_m$, the allowance for regular activities is $\hat{g}_m^{11/5}$, and the allowance for forced activities is $\exp(-\mathrm{monitor}_m^{1/5})$ (`YM/RG/History.lean` lines 80–87) Source . The slope in (1) is the one-loop coefficient of asymptotic freedom (4-06).

Stage 2 One step

One step is a **probability kernel** from a fine configuration U to a coarse configuration V Source . It is not a deterministic map. The kernel has the following form (`YM/RG/Block.lean` lines 53–72) Source .

$$K_\beta(U,V) = \sum_t \ell(U,t) \prod_b \frac{e^{-\beta\, c_{\mathrm{pen}}\, \mathrm{price}(V_b\, Q_b(U,t)^{-1})}}{Z_\beta}. \tag{2}$$

The sum is over choices t (a gauge frame at each coarse lattice point and a predictor label for each coarse link), and $\ell(U,t)$ is the probability of that choice. $Q(U,t)$ is the **predictor** of the coarse links, c_{pen} is the penalty coefficient (`kernel.penalty` in the source's wording), $\mathrm{price}(u) = 1 - \mathrm{Re}\,\mathrm{tr}\, u/m$ is the distance from the identity, of the same form as the Wilson action (`YM/Gauge/MatrixGroup.lean` line 112), and Z_β is the normalisation. The coarse links are scattered in Gaussian fashion around the predictor Inferred . The push-forward is $\mathrm{push}_\beta F(V) = \int K_\beta(U,V) F(U)\, dU$ (line 77) Source . Averaging over the choices with probabilities gives a gauge fixing that keeps the symmetry of the lattice (assessment report § 6.3) Assessment .

Stage 3 Representation and induction

The density of the theory with cut-off N and volume p, pushed forward to step $m \le N$, is held in the following form (`YM/RG/History.lean` lines 165–179) Source .

$$\mathrm{eval}_m(U) = \exp\Big(v\,|\Lambda_m| - A_m(U) - \sum_X E_X(U)\Big)\cdot \mathrm{Gas}_m(U). \quad (3)$$

A_m is the action of the path (the action truncated at the threshold), E_X are the regular activities, and Gas_m is the gas of **forced activities** that exactly covers the marks of the faces exceeding the threshold Inferred . The **induction hypothesis** (`History.Conv` , line 213) is that at each step this representation stays within the calendar's allowances and the coupling lies in $[\hat\beta_m/2,\ 2\hat\beta_m]$ Source . The **one-step relation** (`Transfer` , line 223) consists of the following two Source .

$$\mathrm{eval}_m = \mathrm{push}_{\beta_{m+1}}\,\mathrm{eval}_{m+1}, \qquad \Big|\beta_{m+1} - \beta_m - \frac{11C_{\mathfrak g}}{24\pi^2}\log b_m\Big| \le \mathrm{drift}_{m+1}. \quad (4)$$

The first is an identity, not an approximation Source . So the partition function is preserved at every step (`History.partition_eq` , line 242) Source . The all-steps statement `History.Selected` (line 234) bundles together: step N is the bare Wilson measure; the induction hypothesis and the one-step relation at every step; the coupling equals $\bar\beta$ at step 0; and the running of the coupling Source . The one-step theorem is written in the form "a curve of representations, continuous on an interval of couplings" (`YM/PCalc/Band.lean` line 65) Source . Its proof is the subject of Part 11, and this book has read only part of it.

Stage 4 Shooting

Inducting downward from step N to 0 produces an arc of representations over an interval of bare couplings Source . The ends of the band are

$$\mathrm{low}_m = \tfrac{3}{4}\bar\beta + \tfrac{5}{8}\,s\log D_m, \qquad \mathrm{high}_m = \tfrac{5}{4}\bar\beta + \tfrac{3}{2}\,s\log D_m, \qquad s = \frac{11C_{\mathfrak g}}{24\pi^2} \quad (5)$$

(`YM/FullVolume/Arc.lean` lines 21–22) Source . At step 0 the coupling along the arc has ends $\frac{3}{4}\bar\beta$ and $\frac{5}{4}\bar\beta$, so by the intermediate value theorem a bare coupling can be chosen for which the step-0 coupling is exactly $\bar\beta$ Source . In this way the coupling at spacing 1 takes the same value for all N, p (assessment report § 11) Assessment . The one-step theorem is carried in a form continuous in the coupling in order to use this intermediate value theorem Inferred .

Stage 5 Calibration

For each label of degree d, a reference step $r(d)$ independent of the cut-off is fixed Source . The renormalisation transport induces an invertible linear map on the space of coefficients of local polynomials Assessment . The insertion coefficients at cut-off N are fixed through its inverse, so that "when carried to step $r(d)$ they are exactly the label coefficients" (`calibrated` at line 62 and `normalized` at line 65 of

`YM/Source/Construction.lean`) Source . There is no explicit formula for the entries of the inverse matrix (assessment report, Appendix A) Assessment .

03

Stages 6–10: limits and reconstruction

Stage 6 Ultraviolet limit at fixed volume

For each volume, convergence along the whole sequence of spacings, with kernels and differences (`realUV` at line 45 of `YM/FullVolume/Fields/Filled.lean`) Source . Its form is the implication `positioned` (`YM/FullVolume/Profiles.lean` line 42), which derives locally uniform convergence with differences from uniform boundedness on separated configurations and pointwise Cauchy property at general position Source . The Cauchy property is obtained by combining convergence in the cut-off of the objects at a fixed coarse step j with an estimate that removes step j as $j \to \infty$ Inferred . The latter uses the separation of the points, so the test functions avoid coinciding points Inferred . The source of convergence is not a quantitative comparison estimate but compactness, uniqueness of limits and the subsequence principle Inferred (11-10).

Stage 7 Infinite volume

Reflection positivity is stated for the Wilson measure of the original lattice, for both planes through lattice points and planes through midpoints of edges, along all four axes ($\beta \geq 0$) Source . From it follow the chessboard inequality and the triangle inequality of its root (the root being a seminorm) Source . The main input from the ultraviolet, `Wcmain`, is that the size of renormalised observables measured by the chessboard root is bounded, uniformly in all volumes, as follows Source .

$$\mathrm{root}(O_d) \leq A\, a_m^{-d-\varepsilon} \qquad (m \geq r(d),\ N \text{ sufficiently large, all } p). \tag{6}$$

From this and the representation by the transfer operator follow local boundedness and equicontinuity, and Arzelà–Ascoli gives limits along subsequences of volumes (`YM/Extraction/Lipschitz.lean` line 166) Source . `Wcmain` is not the only input from the ultraviolet; other fields are also passed along the same trajectory Source . The hypotheses of `first_surface` (`TransferLip`, `Bound`) are upper bounds and rates of continuity, and contain no ingredient for uniqueness of the limit Source .

Stage 8 Euclidean invariance and phases

The lattice has only finitely many symmetries, so continuum Euclidean invariance is shown separately. § 6.3 of the assessment report lists three parts Assessment : that the gauge fixing keeps the lattice symmetries; a divergence-free stress tensor and the local Ward identity; and that the stress flux vanishes fast enough at infinity so that

invariance is not spontaneously broken. After that, the commutative algebra of operators compressed to the zero-energy space is given a Gelfand representation, yielding a space of phases X and a probability measure μ (`phaseMeasure` in the source; ν in equation (12) of the assessment report) Source .

$$S^{\mathrm{mix}} = \int_X S^{\xi}\, d\mu(\xi). \tag{7}$$

The proof that the algebra is commutative uses rotational invariance as a hypothesis, so the order is "Euclidean invariance → phase decomposition" Source .

Stage 9 Reconstruction

Each phase S^{ξ} satisfies a variant of the OS axioms including clustering, and a Wightman theory is reconstructed from it Assessment . In each phase the zero-energy space is one-dimensional (uniqueness of the vacuum, without a rate) Source . The chosen phase ξ_* gives the Wightman theory of the public theorem (12-08).

Stage 10 Evidence

The stress tensor lies in a one-dimensional space of conserved symmetric tensors Source . The two-point function of the centred trace, multiplied by $r^8 \log^2(1/r)$, approaches $3 \dim \mathfrak{g}/\pi^4$ Source . Along one sequence of scales, the stress correlations of the mixed theory (the average over phases) approach those of the free Maxwell field Assessment . The coefficients of the operator product expansion do not depend on the state, and the remainder is bounded uniformly in the state Source . The details are in Part 13.

04

What connects the stages

Here are four interfaces that hold the skeleton together.

- **The push-forward identity** (stage 3): because the first relation in (4) is an identity, the representation at step 0 is exactly the push-forward of the original Wilson measure Source . Approximation errors are carried as activities inside the representation Inferred .
- **Curves continuous in the coupling** (stage 3 → stage 4): because the one-step theorem has a form continuous in the coupling parameter, the intermediate value theorem can be used after all the steps Inferred .
- **The root seminorm** (stage 6 → stage 7): estimates uniform in the volume are handed over in the language of the chessboard root built from reflection positivity of the original lattice Source . On the renormalisation side too, the one-step ledger is

a pointwise sum and does not depend on the volume Inferred . What involves an exponent in the volume are estimates at the level of densities and partition functions Inferred .

- **The endpoint is correlations of local fields** (stages 5 and 6): because the endpoint is correlations of local fields at separated points rather than the coarse lattice measure, calibration of observables (stage 5) and a step that removes the coarse step j (stage 6) are needed Inferred .

05

The size of the code and its distribution over layers

The source consists of 9,477 files and about 990,000 lines Source . There are 250,319 declarations, including automatically generated ones, of which 100,168 enter the dependencies of the 17 headline theorems Assessment . Assigning the locations (directories) roughly to the stages of Table 1 and counting gives Table 2.

Table 2 Lines by layer, divided by unit of location (the contents of a location are mixed, so the proportions are a guide Inferred ; the numbers are a count over the map of locations)

Layer	Stage	Locations	Files	Lines	Share
Public statements and assembly	—	10	97	10,071	1.0%
Foundations (groups, lattice, measure)	1	20	133	16,449	1.6%
Definition of one step	2	15	88	12,028	1.2%
Multistep induction and estimates	3, 4	22	6,223	696,999	69.9%
Ultraviolet at fixed volume	5, 6	3	1,184	97,171	9.7%
Infinite volume	7	4	39	4,607	0.5%
Reconstruction	8, 9	12	249	40,167	4.0%
Local fields, operator product expansion, trace	10	14	1,253	100,546	10.1%
Comparison with perturbation theory	10	2	148	12,726	1.3%
Unused locations	—	2	63	6,299	0.6%

The line counts in Table 2 count one extra line at the end of each file (total 997,063; counting line endings gives 987,586) Source . Three things can be read from it Inferred .

- About 70 per cent of the weight of the proof is in the multistep induction and estimates (stages 3 and 4). Part 11 corresponds to this.
- The definitional layer is shallow and short. `YM/RG/History.lean`, which fixes the representation of the induction, is about 250 lines (251 lines) Source .
- The infinite-volume locations are small and shallow. The heavy input comes from the uniform estimates on the ultraviolet side.

06

A guide to the chapters of Volume 2

The right-hand column of Table 1, arranged by part in reading order.

- **Part 8 From the lattice to the continuum — objects**: the sequences of spacings and volumes of stage 1, labels and renormalised insertions (the definitional side of stage 5), the order of limits. From 8-01.
- **Part 9 One step of renormalisation**: the probability kernel of stage 2, and the form of the representation of stage 3. From 9-01.
- **Part 10 Matching the coupling, and asymptotic freedom**: the reasons for the calendar of stage 1, one step of the coupling in stage 3, the shooting of stage 4. From 10-01.
- **Part 11 Multistep estimates and large fields**: the content of the induction of stage 3, and the fixed-step convergence of stage 6. From 11-00.
- **Part 12 Infinite volume and reconstruction**: stages 7–9. From 12-01.
- **Part 13 Evidence that it is Yang–Mills**: stage 10. From 13-01.
- **Part 14 Comparison**: a comparison with this book's Lean, stage by stage. From 14-01.

Part 7 (constructive field theory) is the historical background of this skeleton. § 4.3 of the assessment report says that stages 2–4 follow Bałaban's approach, with many improvements Assessment .

07

What this book's study had seen

This book's study divided the stages a proof must contain into seven (16-09). The set-up "ultraviolet limit at fixed volume → infinite volume by compactness" runs in the same direction as the published construction. The differences stage by stage are as follows (details in 14-01).

- **Stages 1 and 4**: this book took the sequence of bare couplings as given from outside. There was no anchor fixed by matching (fixing the step-0 coupling). Centred blocks with odd sides were a direction this book's study had also tried (the side-3 design, 14-02). Growing the sides with the step was not done.
- **Stage 2**: this book's block map is a deterministic map, and the push-forward measure is its image (19-04). It is not a probability kernel.
- **Stage 3**: this book's ultraviolet Lean goes only as far as theorems at one scale (one step) (the second half of Part 15), and uniform estimates across steps remain

premises (28-04).

- **Stage 6**: this book's chain had the form of subsequential limits. The published construction has convergence of the whole sequence in the spacing.
- **Stage 7**: reflection positivity and the Chessboard Estimate are also in this book (20-01, 22-01). This book has reflection positivity for planes through lattice points and for midplanes. The triangle inequality of the root (the root being a seminorm) is not in this book's catalogue of firm theorems.
- **Stages 8–10**: this book's Lean has no phase decomposition, reconstruction or stress tensor in the continuum limit. What it has is reconstruction and the mass gap on the lattice inside the strong-coupling window (20-04).

08

What has not been read

- The core of the proof of the one-step theorem (located at `Mallow.faces`, line 41 of `YM/PhysicalNext/RG_NEXT_COMPLETE_REPEATABLE_NONLINEAR_STEP/Mallow/Pediment.lean`). We read only the statement.
- The construction that, after one step, puts the activities back within the allowances of the next step, and the lemma on the loss of Haar measure.
- The mathematical meaning of the conditions for choosing the run (the fields of `Funded`). We infer that they are numerical conditions Inferred .
- The body of the transfer-operator estimates (inside `Time.powering` in `YM/Slice/Calendar.lean`), the line of argument that uses rotations in proving commutativity of the phase algebra, and how uniqueness of the vacuum is extended to every phase.
- For many of the large-field lemmas, following the dependency table leads to a different theorem as the headline. Whether they lie on the path of the main theorem has not been checked, except for those confirmed along the chain (from the shooting's `rodB` to the one-step induction, from `realUV` to the synthesis of the run, and so on).
- The assignment to layers in Table 2 is by unit of location; the distribution within a location was not measured.

What this chapter can say

Can say	Cannot say
Source The declarations of Table 1 are at those locations in those forms. (1)–(7) are readings of the source's definitions and statements	Correctness of the proofs (at the level of the assessment). The core of the proof of the one-step theorem
Source The first one-step relation is an identity, and the partition function is preserved at every step	That, because it is an identity, this book's premises fall away
Source Shooting matches the step-0 coupling to $\bar{\beta}$ using the Arc band (5) and the intermediate value theorem	Explicit values of constants that satisfy the calendar's conditions
Inferred About 70 per cent of the code is multistep induction and estimates	The exact distribution within locations

Sources and reproduction

Item	Type	Source / tool
Stages 1–4	Source	OpenAI-YM-20261028: `YM/Lattice/Scale.lean` lines 28–44 and 128–129, `YM/RG/History.lean` lines 64–87 and 165–250, `YM/RG/Block.lean` lines 53–77, `YM/PCalc/Band.lean` line 65, `YM/FullVolume/Arc.lean` lines 21–22 and 95, `YM/FullVolume/Stage/Rods.lean` line 26
Stages 5, 6	Source	`YM/Source/Construction.lean` lines 44–65, `YM/FullVolume/Profiles.lean` lines 25, 42, `YM/FullVolume/Fields/Filled.lean` line 45
Stages 7–9	Source	`YM/Slice/Reflection.lean` line 98, `YM/Cells/Chess.lean` lines 80, 108, `YM/Extraction/Center.lean` lines 38–41, `YM/Extraction/Lipschitz.lean` line 166, `YM/Phase/Mixture.lean` line 127, `YM/EveryPhase/Vacuum.lean` line 146, `ChallengeSpec.lean` line 117
The three parts of stage 8; stages 2–4 and Bałaban's approach	Assessment	Assessment report § 4.3, § 6.3
Common unit, calibration, number of declarations	Assessment	Assessment report § 9.1, § 11, Appendix A
Line counts	Source	Lines of `.lean` files counted on our machine (9,477 files, 987,586 lines). The distribution over layers is a count over the map of locations

Words used in this chapter — calendar, probability kernel, predictor, penalty coefficient, induction hypothesis, one-step relation, truncated action, regular activity, forced activity, shooting, calibration, root seminorm, phase decomposition. Their meanings are in the A-02 Glossary.

Next chapter to read: 6-04 What is not shown — where this skeleton does not reach, set out from the assessment report's five open questions and the source's "what it does not say".

What is not shown — a bridge to Volume 3

> This chapter is existing knowledge (a reading of the published construction). § 08 is this book's own study, and the "not shown" side of § 01– § 07 is a map of the unsolved.

What this chapter shows — Nine items the published construction does not reach. For each, what the assessment report says, how far the source goes, and where it stops. What the polynomial estimates that stand in place of a mass gap say and do not say. At which item's opposite end (strong coupling, fixed lattice spacing) the results of this book's Volume 3 lie.

Prerequisite chapters — 6-01 (the four requirements), 6-02 (the three targets), 6-03 (the skeleton).

Said first — "Not shown" does not mean "false". The committee writes that the fact that uniqueness is not shown does not contradict the general expectation that it holds Assessment . Much of what we describe as "not in the source" is a judgement within the range of our searches. All the results of this book's Volume 3 are statements at a fixed lattice spacing, and this book shows none of the items listed here.

The order of this chapter

1. Overview — nine items and where they stand (Table 1)
2. Convergence and uniqueness — subsequences of volumes, and quantities that may depend on the subsequence
3. A probability measure for the continuum theory — only the table of correlations and the OS space
4. A uniform mass gap — what exists are polynomial estimates
5. Wilson loops and the area law, one-particle states — outside the local labels
6. Sequences of spacings, coinciding points, perturbation theory — universality, contact terms, agreement to all orders
7. Fermions — a construction of pure Yang–Mills
8. A bridge to Volume 3 — where this book's results stand, item by item (Table 2)
9. What has not been read

Labels: Source = we read the source of the published construction / Inferred = this book's inference / Assessment = from the committee's assessment report. The other labels (Proved here, Cited, Reading, Lean and so on) mean the same as in the rest of the book (see "Labels" in 6-00). Unless stated otherwise, paths are relative to the root of the repository `OpenAI-YM-20261028`.

01

Overview

§ 8 of the assessment report lists the main points the present construction does not address as five open questions Assessment .

1. Show convergence rather than subsequential convergence, and show that the infinite-volume limit is unique.
2. Define a probability measure for the continuum theory on a suitable space, such that it gives the Schwinger functions of the construction.
3. Show a uniform mass gap, including exponential decay of the corresponding correlations and a gap in the spectrum of the OS Hamiltonian.
4. Construct Wilson-loop observables and relate them to the local observables of the construction. Show the area law for Wilson loops.
5. Construct one-particle states and study their energy–momentum spectrum. In many constructive models in two and three dimensions, this analysis has been carried out.

Table 1 adds to these five four more items, from the body of the assessment report and from the source's "what it does not say".

Table 1 Nine items not shown (the right-hand column lists related chapters of Volume 3)

Item	Assessment report	In the source	Not in the source	Volume 3
Convergence and uniqueness	§ 8 item 1, § 5, § 6.2	Limits along subsequences of volumes; uniqueness of the limit along the same subsequence	Convergence in the volume; independence of the subsequence	16-03, 20-03
Probability measure	§ 8 item 2, § 3	Table of correlations and OS space	Continuum measure	16-02
Uniform mass gap	§ 8 item 3, § 2	Polynomial estimates uniform in each phase	Exponential decay, spectral gap	20-04, 16-04
Wilson loops and the area law	§ 8 item 4	Parallel transport along lattice paths	Continuum loop observables, area law	20-06, 21-03

Item	Assessment report	In the source	Not in the source	Volume 3
One-particle states	§ 8 item 5	—	One-particle states, mass shell	16-02
Universality in the sequence of spacings	§ 4.3, § 9.4	Convergence along the whole of the one sequence of spacings chosen by the family	Convergence along other sequences of spacings	—
Where points coincide	§ 7.5, § 10	Correlations at separated points; existence of an extension with contact terms	Uniqueness of the extension, OS positivity, convergence from the lattice	—
Agreement with perturbation theory to all orders	§ 7.4, § 7.5	One-loop running of the coupling; coefficient of the leading term of the trace	Agreement at every order	—
Fermions	(not treated)	—	Matter fields	20-05

02

Convergence and uniqueness

The volume limit is taken only along subsequences, by Arzelà–Ascoli compactness (assessment report § 5)[Assessment] . It is shown that one subsequence works for all correlations simultaneously[Assessment] . Whether the limiting correlations depend on the chosen subsequence is open[Assessment] .

On the source side, what goes into the theorem that takes the volume limit, `first_surface` (`YM/Extraction/Lipschitz.lean` line 166), is only upper bounds for local boundedness and equicontinuity[Source] . Statements that could serve as material for uniqueness — such as the difference when the volume is changed, decay of correlations, or the vanishing of dependence on boundary conditions — are not among them[Source] . The uniqueness stated by `Order.unique` (`YM/Extraction/Orders.lean` line 70) is uniqueness of the limit along the same subsequence, that is, uniqueness of limits in a Hausdorff space[Source] . This lemma is outside the dependencies (outside) and is not reached from any headline[Source] .

There are other quantities that may depend on the subsequence[Assessment] .

- Whether the components of the phases are equal to one another. Each component has clustering, but clustering of the mixed theory S^{mix} is not shown (§ 6.2). The assessment report writes that if the mixture had clustering all components would be equal, but that this would still not suffice for uniqueness of the infinite-volume limit.

- That the coefficients of the operator product expansion do not depend on the choice of subsequence of volumes (§ 7.5).
- That the short-distance effective coupling $g_W(r)$ does not depend on the subsequence of volumes (§ 7.4).

The spacing side is different. The ultraviolet limit at fixed volume converges along the whole sequence of spacings chosen by the family Source . Subsequences remain only on the volume side (6-02 Table 3).

03

A probability measure for the continuum theory

§ 3 of the assessment report writes that defining, on a suitable space, a probability measure corresponding to the formal expression $Z^{-1}\exp\!\big(-\frac{1}{2g^2}\int\sum_{\mu<\nu}\|F_{\mu\nu}\|^2\big)\,\mathcal{D}A$ is not something the submission claims Assessment . The continuum objects constructed are the correlation functions of local observables and the quantum fields reconstructed from them Assessment .

On the source side too, the continuum objects are the table of correlations and the OS Hilbert space, not a measure Inferred . For the intermediate objects of renormalisation as well, the effective density at a fixed step m is Cauchy uniformly in the cut-off, but its limiting measure is not defined as a named object Inferred . Reflection positivity of the push-forward measure is not stated either Inferred , because the patches of the one-step kernel straddle the reflection plane Inferred .

04

A uniform mass gap

The construction gives no evidence, for or against, about the mass gap (assessment report § 2) Assessment . What § 8 item 3 asks for is exponential decay of correlations and a gap in the spectrum of the OS Hamiltonian H Assessment .

What the source has are the following two polynomial estimates, uniform in each phase Source .

$$\sigma_A\big((0,u]\big)\le M\,u^{2}\quad(0<u\le 1),\qquad 0\le\big\langle A,\,(e^{-\tau H}-P_0)\,A\big\rangle\le M\,\tau^{-2}\quad(\tau\ge 1).\tag{1}$$

The left is the weight at energies $(0,u]$ of the spectral measure σ_A (named `vletter` in the source) of a vector A built from finitely many words (`light_bound` in `YM/EveryPhase/Light.lean`, line 221) Source . The right is a heat estimate, where τ is Euclidean time and P_0 is the projection onto the zero-energy space (`inverse_square` in

`YM/EveryPhase/Decay.lean`, line 173) Source . Both are used in the dependency table, and the first headline is `YM.SharpGram.habitat` Source . Whether they reach the public theorem or the construction theorem has not been checked.

If there were a mass gap Δ, the weight on the left would be 0 for $u < \Delta$, and the right would decrease like $e^{-\Delta\tau}$. (1) is no substitute for that, because its form is compatible with theories that have massless particles Inferred . We read the origin of (1) as local commutativity and the forward light cone, not an input specific to Yang–Mills Inferred . Put differently, the mass gap question is the question "make the weight in (1) vanish below some Δ" Inferred .

One more point about the edge of the construction's scope. The renormalisation steps stop at step 0, at spacing 1, where the coupling reaches the reference value $\bar{\beta}$ Source . What can be read from the source about $\bar{\beta}$ goes only as far as the calendar's conditions giving $\hat{\beta}_m > 1$ (`fits.delayed`, used at line 142 of `YM/PertExtension/Running.lean`) Source . How large it is taken is inference Inferred . The long distances beyond that are handled not by renormalisation but by reflection positivity, the chessboard and Arzelà–Ascoli Source . The mass gap question can probably be restated as the question of following this lattice theory at spacing 1 out to long distances Inferred . To restate it that way, three things are needed first: the limiting object, local estimates uniform in the volume, and the image of the observables (16-03).

05

Wilson loops and the area law, one-particle states

Parallel transport along lattice paths is in the source's foundations (`travel` in `YM/Lattice/Path.lean`, line 105) Source . However, we have not found Wilson-loop observables in the continuum limit or a statement of their area law Source (within the range of name searches). Across all `.lean` files, the words area law, area_law, Wilson loop, wilson_loop and WilsonLoop occur 0 times. The word loop occurs often, but as in one-loop coefficients or the calendar's `loopCap` Inferred . An example among the committee's checks writes that the labels are polynomials in the curvature at a point, and that completeness is not shown for non-local observables such as line defects (`submissions/openai-20261028/docs/examples/OperatorContent-20261028.lean` lines 35–38) Source . § 8 item 4 asks not only to construct Wilson loops but to relate them to the local observables Assessment .

We have also not found in the source one-particle states of the interacting theory or an analysis of their spectrum Source (within the range of name searches: one particle, single particle, oneParticle). The words occur in only one place, the docstring "One particle test vectors" at line 4 of `YM/TraceExtension/Fock/Weyl.lean` Source . Since it is in the Fock location, we infer that it belongs to the Fock space of the free field Inferred .

Separately, the docstring at line 5 of `YM/TraceExtension/Damping.lean` has the heading "states of a one-point insertion and their spectral representation", and the same docstring says it does not assume a mass gap Source . § 8 item 5 writes that this analysis has been carried out in many constructive models in two and three dimensions Assessment (7-03).

06

Sequences of spacings, coinciding points, perturbation theory

Universality in the sequence of spacings

The sequence of spacings a_N is chosen by the family. The convergence is along the whole of that chosen sequence Assessment . § 4.3 of the assessment report writes that there is no universality result saying that every kind of sequence of spacings gives the same limit Assessment . The construction's sequence is a special one, whose sides b_m are odd numbers at least 3 that grow with the step Source . We read growing the sides as a choice made to remove the one-step error by dividing it by $\log b_m$ Inferred (10-05).

Where points coincide

Correlations are defined only on test functions on configurations of mutually distinct points Assessment . Using the operator product expansion, the correlations can be extended to test functions that include coinciding points Assessment . However, uniqueness of that extension, OS positivity on unrestricted Euclidean test functions, and convergence from the lattice are not shown (§ 7.5) Assessment . The committee's independent OS matching uses test functions all of whose derivatives of every order vanish on every diagonal, so contact terms supported on the diagonal are not seen (§ 10) Assessment .

Agreement with perturbation theory to all orders

Agreement with perturbation theory at every order is not shown (§ 7.5) Assessment . The truncation degree D of the operator product expansion is the degree of the labels, not the loop order Assessment . The committee's matching neither computes the expansion coefficients independently nor compares them with perturbative coefficients (§ 11) Assessment . The agreement that is stated is at the level of leading terms: the one-loop running of the coupling (6-01 Table 1) and the coefficient of the short-distance law of the trace Assessment . There are three declarations of a form close to the short-distance effective coupling (§ 7.4): `Running.ratio` is outside the dependencies, `wilson_link` is an intermediate step inside the dependencies, and the headline `Asymptotics.comparison` is the step coupling with cut-off, a different quantity from g_W ; none survives in the conclusions of the three targets (6-02) Source .

07

Fermions

The construction is of pure Yang–Mills theory. The lattice measure of the public theorem is Haar measure on the link variables weighted by the Wilson action, and has no matter fields (`wilsonLaw` in `PublicWilson.lean`, lines 29–33) Source . Searching the source's `.lean` files for the words fermion and Grassmann found only one occurrence, in a docstring in the free-field Wick location: "no signs from a fermionic grading" Source . A construction of a theory with quarks (quantum chromodynamics) is outside the scope of this construction Inferred .

08

A bridge to Volume 3

Volume 3 treats the infrared side of the items listed here (uniqueness, the mass gap, the area law). All of this book's results are statements at a fixed lattice spacing, and show nothing about the continuum theory. They are statements at the opposite end (strong coupling, fixed lattice spacing) from the questions the published construction left, and statements of a kind the published construction does not have. Table 2 lists the position item by item.

Table 2 Items not shown, and the results of this book's Volume 3 (this book's side is all at a fixed lattice spacing)

Item	This book's result	What it does not say
Convergence and uniqueness	Inside the strong-coupling window, an infinite-volume limit without subsequences (convergence of the whole sequence, 20-03)	Weak coupling, the continuum limit
Mass gap	On the lattice inside the strong-coupling window, a gap for the transfer operator (20-04)	Outside the window; a uniform gap in the continuum limit
Area law, confinement	On the lattice inside the strong-coupling window, the area law for Wilson loops and an upper bound on the vortex free energy (20-06). Inside the window there is a form whose rate does not depend on the transverse size (the prefactor and the transverse area remain). The vortex floor in a twisted box (21-02, 21-03)	A form uniform in the transverse size outside the window; the area law in the continuum limit
Dependence on boundary conditions	Ratios of partition functions with twisted boundary conditions, and Polyakov loops (21-06)	Implications for uniqueness of the infinite-volume limit
Rarity of large fields	For the Wilson measure at one scale, rarity uniform in the volume (22-01)	Measures at coarse scales; a form across steps
Fermions	A vessel with the Wilson–Dirac weight (20-05)	The continuum limit, chiral symmetry

The published construction treats only lattices with periodic boundaries and has no statements about twisted boundary conditions or the vortex free energy Inferred . This book's vortex results are quantities that measure dependence on boundary conditions. The chain that would carry them to the continuum theory or to a uniform gap is still left as premises (28-04). The open questions are organised in 16-02, and this book's position in 16-04.

09

What has not been read

- The items described as "not in the source" (continuum Wilson loops, one-particle states, the continuum measure) are judgements within the range of the word searches described in § 05. We have not excluded the possibility of declarations that treat the same content under other names.
- We have not read the proof of (1). That its origin is local commutativity is inferred from the names of lemmas in the proof.
- We have not checked explicit conditions on how large the construction takes $\bar{\beta}$.
- Whether there really is no named statement about the limiting measure at a fixed step has been searched only in part of the locations (under `YM/FullVolume/`).
- We have not read how the "One particle test vectors" of `Weyl.lean` and the "states of a one-point insertion" of `Damping.lean` relate to one-particle states of the interacting theory.

What this chapter can say

Can say	Cannot say
Assessment The assessment report lists five open questions and writes that the construction gives no evidence about the mass gap	That they are false (they are merely not shown)
Source Only upper bounds go into the volume-limit theorem, and uniqueness holds only along the same subsequence	That the limit does not depend on the subsequence
Source There are polynomial estimates (1), uniform in each phase	Exponential decay, a spectral gap
Source The lattice measure has no matter fields	—
This book's results in Table 2 are statements at a fixed lattice spacing	This book shows none of the items of Table 1 for the continuum theory

Sources and reproduction

Item	Type	Source / tool
Five open questions	Assessment	Assessment report § 8
Dependence of subsequences, phases, coefficients, couplings	Assessment	Assessment report § 5, § 6.2, § 7.4, § 7.5
Inputs to the volume limit, and uniqueness	Source	OpenAI-YM-20261028: `YM/Extraction/Lipschitz.lean` line 166, `YM/Extraction/Orders.lean` line 70
Polynomial estimates	Source	`YM/EveryPhase/Light.lean` line 221, `YM/EveryPhase/Decay.lean` line 173
Parallel transport	Source	`YM/Lattice/Path.lean` line 105
Completeness for line observables	Source	yang-mills-group/YM: `submissions/openai-20261028/docs/examples/OperatorContent-20261028.lean` lines 35–38
Probability measure, universality, contact terms, perturbation theory	Assessment	Assessment report § 3, § 4.3, § 7.5, § 10, § 11
Being pure Yang–Mills	Source	`PublicWilson.lean` lines 29–33. Word search (fermion, Grassmann): only `YM/Maxwell/Wick.lean` line 10
This book's results	—	The source tables of each chapter of Parts 20–22

Words used in this chapter — subsequential limit, uniqueness, clustering, mass gap, spectral measure, heat estimate, Wilson loop, area law, one-particle state, universality, contact term, agreement with perturbation theory. Their meanings are in the A-02 Glossary.

Next chapter to read: 7-01 What constructive field theory is — we return to the tools and history of constructive field theory, the background of the published construction. To go on to Volume 3, read 16-02 Five open questions.