Volume 2 Existence — reading the published construction

Part 11 Multistep estimates and large fields

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Entrance — the induction hypothesis and the one-step certificate

This chapter is a guide. § 02 is the convention for levels of citation in Volume 2, and § 05 is this book's own investigation.

What this chapter shows — The shape of the "induction hypothesis" that the published construction keeps at each step, and of the "one-step certificate" it produces at each step, as it stacks renormalisation steps from step N (the bare Wilson measure) down to step 0 (spacing 1). The two carry ledgers of different strength. A table shows which part of this mechanism each of the eleven chapters of this part deals with.

Chapters assumed — 9-01 The block kernel (the one-step probability kernel), 9-06 The density representation, 9-07 The small-field threshold and the truncated action, 10-04 The shooting method, 10-05 Why the sides grow, and the calendar.

Said first — What this part writes is the definitions and statements of the published construction, as far as this book has read them in the source. The certainty of the results of the published construction is at the level "the committee's assessment is that it is proved in Lean"; what this book has checked goes only as far as the form of the definitions and statements. Much remains unread (11-11). This part says nothing about the mass gap or about uniqueness in infinite volume.

The order of this chapter

1. What this part reads — the three components of the induction that descends the steps.
2. Levels of citation and labels — Source Inferred Assessment , and which target a statement enters.
3. The two ledgers — the induction hypothesis is weak, the one-step certificate strong.
4. A map of this part — the eleven chapters and their main declarations.
5. What this book's study had seen — how it differs from stacking up regions.
6. What has not been read — the list is in 11-11.

Labels: Source = read in the source of the published construction / Inferred = this book's inference / Assessment = according to the committee's assessment. The other labels (Proved here, Cited, Reading, Lean and so on) mean the same as in the rest of the book (for details see "Labels" in 6-00).

01

What this part reads

The published construction starts from the Wilson measure on the lattice of spacing $a_N = 1/D_N$, applies renormalisation with block sides $b_{N-1}, \dots, b_0$ in turn, and descends to step 0, of spacing $a_0 = 1$. One step is not a fixed map but a probability kernel (9-01). On the configurations V of the lattice at step m sits an unnormalised density $\rho_{N,p,m}(V)$, and the steps are linked by push-forward identities Source .

$$\rho_{N,p,m}(V) = \int T_{\beta_{m+1}}(U,V)\,\rho_{N,p,m+1}(U)\,dU, \qquad \rho_{N,p,N} = e^{-\beta_N S_W}. \tag{1}$$

Here p is the volume index, T_β is the one-step kernel, and dU is the product of the probability Haar measures. Equation (1) is an identity, not an approximation. What this part reads are the three components for carrying this sequence of identities "with estimates".

1. **Induction hypothesis**: the density at every step has a representation of the same form, and its ingredients carry a ledger for each step (11-01).
2. **One-step certificate**: building the representation at step m from the representation at step $m+1$, and producing equation (1) and the ledgers. It splits into large fields (11-02–11-08) and small fields (11-09).
3. **Convergence at a fixed step**: with step m held fixed and $N \to \infty$, the ingredients of the representation converge along the whole sequence, and from this the lattice correlations converge (11-10).

02

Levels of citation and labels

Volume 2 reads, by citation, the Lean source of the existence construction for pure Yang–Mills published in November 2026 (repository `yang-mills-group/OpenAI-YM-`

`20261028`, Apache-2.0) and the committee's assessment (`docs/assessment-1.pdf` in `yang-mills-group/YM`, 2026-11-10).

Level of certainty. The committee records that the declarations in the dependencies of the 17 headline theorems (about 100,000) were rechecked in a freshly initialised Lean kernel environment (one that loads fixed external versions), and that they use only the three permitted axioms[Assessment] (assessment § 9.1). The public theorem `YangMills.jointExistence` and the committee's own combined target `YMReview.Acceptance.combined_acceptance` have also passed a separate comparison tool (Comparator)[Assessment] (assessment § 9.2). The assessment also states that no separate kernel implementation was used[Assessment] (§ 9.2). Therefore what this part calls "statements of the published construction" are results at the level **the committee's assessment is that they are proved in Lean**. What this book has checked itself goes only as far as reading the form of the definitions and statements in the source. Of the proofs, only the key points have been read.

The labels follow the legend of 6-00: there are three, [Source] (a definition or statement read in the source of the published construction), [Inferred] (this book's inference) and [Assessment] (according to the committee's assessment). [Source] does not guarantee that a proof is correct; that is the level of the committee's assessment.

Which target a statement enters. The verification record (`verification/declaration-usage.tsv`) notes for each declaration whether it enters the dependencies of one of the 17 headline theorems (used) or not (outside). "Used" does not mean "used by the public theorem". 6-00 divides the targets into five (the public theorem, the construction theorem, the committee's combined target, reached from one of the headline theorems, outside the dependencies). Many of the declarations in this part are not the conclusions of the first three themselves but components of their proofs. So in this part we indicate how far we have checked with the following three descriptions. The first is for those we have checked, by tracing the declarations literally, to lie on the proof chain of the construction theorem (and of the public theorem, which contains it as a projection); the second and third are exactly the last two distinctions of 6-00.

Description	Meaning	Examples
On the chain of the existence theorem	Checked by tracing the declarations literally from `Nonperturbative.existence` (called by the construction theorem `constructed`)	`rodB` → `Roof.Lyrics.supplier` → … → `Return.reset`, `Window.allB`, `sailsW`, `realUV`
Reached from one of the headline theorems	"used" in the record. Not checked whether it lies on the chain of the existence theorem	Most of the large-field lemmas (the witness in the record is `SharpGram.habitat`)
Outside the dependencies	"outside" in the record. Not used in the proofs of the main results	`Remainder.hole_price`, `Directed.holding`

Sources are given as "file path (relative to the root of the repository; versions as in the legend of 6-00), declaration name, line". The lines are ones this book checked by opening the source.

03

The two ledgers

Write the reference coupling at step m as $\hat{\beta}_m = \bar{\beta} + s\log(1/a_m)$ ($s = 11C_g/(24\pi^2)$ is the one-loop slope), $\hat{g}_m = \hat{\beta}_m^{-1/2}$, and the super-polylogarithmic quantity as $P_m = \exp\left((\log\log\hat{\beta}_m)^2\right)$ (the calendar `Calendar`, `YM/RG/History.lean:64-90`) Source . The small-field threshold is $t_m = \hat{g}_m P_m$.

The **induction hypothesis** `History.Conv` (`YM/RG/History.lean:213`) requires the following of the step-m representation Source .

- The ledger of regular activities (small-field corrections) $\le \hat{g}_m^{11/5}$ (`allowance`, :84).
- The ledger of forced activities (large-field corrections) $\le \exp(-P_m^{1/5})$ (`forcedAllowance`, :85).
- The coupling is in the band: $\beta_m \in [\hat{\beta}_m/2,\, 2\hat{\beta}_m]$.

The **one-step certificate** `Directed.Move` (`YM/PCalc/Band.lean:49`) states, for a representation v at step $m+1$ and a representation u at step m, the push-forward identity $u.\mathrm{eval} = \mathrm{push}_{\beta_v}(v.\mathrm{eval})$ and the form of the induction hypothesis (the weak ledger), and in addition a **strong** ledger for the forced activities Source .

$$\text{induction hypothesis: } \|K\| \le e^{-P_m^{1/5}}, \qquad \text{one-step certificate: } \|K\| \le e^{-g\,P_{m+1}^{2}}. \tag{2}$$

Here $\|K\|$ is the pointwise sum weighted by coverings (`KBound`, `YM/RG/Activities.lean:102`), and $g > 0$ is a constant the calendar chooses at each step. The strong ledger is rebuilt from the large-field price of that step (the exponent of the factor e^{-cEP^2} that the one-step integral yields per seed; seeds are defined in 11-03, marks in 11-05) (11-05). It passes to the weak ledger by a one-line inequality that lowers the exponent of P (11-07) Source .

The induction hypothesis carries only the weak ledger; within one step a strong estimate is rebuilt and then relaxed to the weak ledger. There is no need to carry the strong ledger as a hypothesis for the next step Inferred . The weak ledger is also a function of step m and is rebuilt at every step. The conditions for making things hold

uniformly in the step are paid for on the side of the calendar's constants (the entry for β, the lower bound on the sides, and so on) (11-08) `Source` .

04

A map of this part

Chapter	Subject	Main declarations (`Source`)	How it sits in the targets
11-01	The form of the density at each step, and the induction hypothesis	`Rep` , `Rep.eval` , `Conv` , `Transfer` , `Selected`	Objects of the chain of the existence theorem (`Makes` contains `Selected`)
11-02	The finest step	`WilsonStartup.due_eq`	Reached from one of the headline theorems
11-03	Decomposing the action, seed types, regions, partitions of unity	`Tube.locates` , `Stores` , `Seeded` , `Plot` , `Allocation`	Reached from one of the headline theorems
11-04	Energy per seed	`Bid.Crowds` , `Seals` , `Lapse` , `Teller.Grants`	Reached from one of the headline theorems
11-05	Allocating prices, and polymer sums	`Buy.quote` , `Cash.armK`	Reached from one of the headline theorems
11-06	Reorganising activities at the coarse step	`ThinP` , `ThickP` , `Collection.vac`	Reached from one of the headline theorems
11-07	Converting between ledgers, and persistence across steps	`Time.late_force` , `Postage.saved` , `Time.now`	Reached from one of the headline theorems
11-08	The calendar — the order in which constants are chosen	`Supply.Large` , `Bid.pricing_order` , `Calendar.Fits`	Reached from one of the headline theorems
11-09	The small-field step	`Return.earnings` , `atlas` , `Local.actual` , `Return.reset`	On the chain of the existence theorem up to `reset`
11-10	Convergence at a fixed step, and the step that removes the step index	`Window.allB` , `accord` , `Flux.fanned` , `Sever.echoes` , `Rising.arrives`	On the chain of the existence theorem
11-11	List of what has not been read	—	—

We recommend reading in the order of the table. Skipping the large-field chapters (11-02 to 11-08) and going from 11-01 to 11-09 and 11-10 still lets you follow the thread.

Correspondence of notation

The chapters of Parts 9–11 were each written in notation close to the letters of the source, so in places the same object has a different letter in different chapters, and in places the same letter denotes different objects in different chapters. We collect the correspondences for reading into one table.

Object	Parts 9 and 10	Part 11	Note
Wilson action	S_{W} (9-06, 10-01)	S_{W} (equation (1)). Equation (1) of 11-02 writes it as the sum of prices over plaquettes, $\sum_{\square} \mathrm{price}(U_{\square})$	The same thing. Both are sums over plaquettes before multiplying by β; the weight multiplies by β and puts it in the exponent (S_W in A-01 § 05 is written in the form that includes β)
Small-field threshold	t_m (step m), t_{m+1} (step $m+1$) (9-07)	Depending on the chapter, t_m, τ, t, t_y	The table correspondence of threshold letters in 11-01
Covering scale	$B_m = \lceil (\log \hat{\beta}_m)^2 \rceil$ (9-06, 9-07)	B_m (11-01, 11-08, 11-10), $R = \mathrm{radiusB}(x)$ (11-03, 11-04; equal to B_m at $x = \hat{\beta}_m$)	The $R = (J+3)L$ of 9-02–9-04 is the radius of a patch, a different quantity
Discrete choice of the kernel (a pair of frame label and path label)	τ (9-01–9-05: $P(\tau \mid U)$, $Q_b(U, \tau)$)	k (11-03, 11-04: $Q = Q(U, k)$)	The k of 11-06 is a fusion index, a different quantity
Linearisation of the predictor inside the precision matrix	Q (9-01: $K + c_{\mathrm{pen}} Q^{\mathsf{T}} Q$)	Π (11-09: $K + c_{\mathrm{pen}} \Pi^{\mathsf{T}} \Pi$)	The same matrix (`FreeResolvent.baseMatrix`). The Π_G of 9-04 is the projection to the group, a different thing
Dimension of the representation	d_G (9-01, 10-01)	d (11-01, 11-02)	The d of 11-05 and 11-10 is a degree, a different quantity
Correction to the coupling (deviation from the one-loop prediction)	d ((1) of 10-02, the root of the rereading equation in 10-03)	$u = \beta_F - \mathrm{predictor}$ and Δ (11-09)	The same quantity (11-09 § 06)
Coefficient of the strong ledger of the one-step certificate	—	g (equation (2), 11-08), d (11-07; the letter in the source's `late_force`)	The same coefficient. The g_m of 11-10 is the one-step deviation of the coupling, a different quantity

Object	Parts 9 and 10	Part 11	Note
Names of the fields of the representation	main action, truncation parameter, ledger point, support (9-06)	path action, truncation constant, budget point, support (11-01 onward)	The same things

05

What this book's study had seen

This book's study treated the ultraviolet side as "a chain of block maps" and thought in terms of stacking up large-field regions step by step and fixing them thereafter (the Bałaban type). What was open there was how to handle defects that persist over many steps near configurations whose shape the block map does not change (the "large-field estimate" and "large-field gas" in the premise list, 28-04, 28-06). What this book had in Lean went only as far as the rarity of large plaquettes at a single scale (22-01) and one step of the free field (15-12).

The published construction **answers this obstacle in a different form**. It keeps no history of regions; it recounts seeds at every step and, with an inequality uniform over configurations satisfying the gauge section and the region conditions, produces a factor per seed at every step. It does not change the measure (equation (1) stays an identity) Source (form read) / Inferred (the reading of how this corresponds to this book's obstacle). The full comparison is in 14-04.

06

What has not been read

At the end of each chapter of this part, we list, with locations, what has not been read on that chapter's subject. The list for the whole part is 11-11. There are three large items.

- The whole of the upper bounds on the integrals that remain after the large-field price has been extracted (from `YM/SectorNext/LARGE_FIELD_REMAINDERS` onward).
- The content of how the one-step theorem is assembled (`Use.Machine.run`, `YM/AnalyticBridge/VERIFIED_RESET_STEP/Downstream.lean:65`).
- The proofs of the lemmas that reorganise the activities at the coarse step (inside `YM/PhysicalNext/RG_NEXT_COMPLETE_REPEATABLE_NONLINEAR_STEP`).

What this chapter can say

Can say	Cannot say
Source The steps are linked by push-forward identities, and the induction hypothesis carries the weak ledgers $e^{-P_m^{1/5}}$ and $\hat{g}_m^{11/5}$ and the coupling band (`Conv`, `Transfer`)	That the representation is unique (the source does not claim uniqueness)
Source The one-step certificate states the strong ledger $e^{-gP_{m+1}^2}$ for forced activities (`Move.forced`)	How the strong ledger is used at later steps (not read)
Assessment The dependencies of the 17 headline theorems were rechecked in a freshly initialised kernel environment (assessment §9.1; no separate kernel implementation was used, §9.2)	That this book has checked the proofs of the lemmas in this part
Inferred The strong ledger is not carried as a hypothesis for the next step; it is rebuilt within a step and then relaxed to the weak ledger	That no condition uniform in the number of steps is needed (it is paid for on the calendar's side)

Sources and reproduction

Item	Kind	Source / tool
Lean source of the published construction	Source	`yang-mills-group/OpenAI-YM-20261028` (Apache-2.0). This book only read a copy; it neither built nor ran it
The committee's assessment	Assessment	`docs/assessment-1.pdf` in `yang-mills-group/YM` (2026-11-10), §4.3, §9.1, §9.2. Versions as in the legend of 6-00 (code 3456e5e, assessment side 55ba9fc)
Calendar and ledgers	Source	`YM/RG/History.lean:64` (`Calendar`), :84 (`allowance`), :85 (`forcedAllowance`)
Induction hypothesis, one-step relation, all steps	Source	`YM/RG/History.lean:213` (`Conv`), :223 (`Transfer`), :234 (`Selected`)
One-step certificate	Source	`YM/PCalc/Band.lean:49` (`Directed.Move`), :65 (`Program`)
Dependency record	Source	`verification/declaration-usage.tsv` (the status and headline-witness columns)

Words used in this chapter — induction hypothesis, one-step certificate, ledger, regular activity, forced activity, push-forward identity, headline theorem. Their meanings are in the A-02 glossary.

Next chapter to read: 11-01 The form of the density at each step — we read, in formulas, the representation that the induction hypothesis carries.

2026-11-13 · chapter Yang–Mills and the mass gap Part 11 Multistep estimates and large fields The density representation ·
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The form of the density at each step, and the induction hypothesis

> This chapter is established knowledge (a reading of the published construction). § 04 is this book's own investigation and § 05 is a map of open questions.

What this chapter shows — As what function of what ingredients the unnormalised density at step m is written, and what ledgers those ingredients carry. The content of the induction hypothesis `History.Conv`, the one-step relation `History.Transfer`, and the statement for all steps `History.Selected`.

Chapters assumed — 11-00 (labels and levels of citation), 9-06 The density representation, 9-07 The small-field threshold and the truncated action.

Said first — The representation is stated by an existential quantifier and is not unique (the source's docstring says so explicitly). The ledgers are per-step quantities in the form of pointwise sums independent of the volume, but the size of the density itself includes an exponential in the volume. This chapter only reads the form; the proof that the ledgers are preserved is the subject of 11-02 to 11-09.

The order of this chapter

1. Statement — the representation, the induction hypothesis, the one-step relation, all steps
2. The truncated action and the path action — linear above the threshold.
3. The two kinds of activity, and the gas — regular activities only where healthy, forced activities only on the flags.
4. What this book's study had seen
5. What has not been read

Labels: Source = read in the source of the published construction / Inferred = this book's inference / Assessment = according to the committee's assessment. The other labels (Proved here, Cited, Reading, Lean and so on) mean the same as in the rest of the book (for details see "Labels" in 6-00).

01

Statement — the representation, the induction hypothesis, the one-step relation, all steps

Notation follows the calendar (11-00 § 03): $\hat{\beta}_m$, $\hat{g}_m = \hat{\beta}_m^{-1/2}$, $P_m = \exp((\log\log\hat{\beta}_m)^2)$, the threshold $t_m = \hat{g}_m P_m$, and the covering scale $B_m = \lceil(\log\hat{\beta}_m)^2\rceil$. Let Λ_m be the lattice at step m and V a configuration on it.

Correspondence of threshold letters. Within one step (from step $m+1$ to step m) two thresholds appear, one on the fine side and one on the coarse side (9-07). The letters differ from chapter to chapter, but each is the value of the calendar threshold $t_m = \hat{g}_m P_m$ at step m or at step $m+1$. In the source, `temp` (`YM/Pressure/Seeds.lean:25`) is $\mathrm{temp}(\hat{\beta}_m) = t_m$, and `fiery` (:26) is $\mathrm{fiery}(\hat{\beta}_m, L) = \mathrm{temp}(\hat{\beta}_m + s\log L) = t_{m+1}$ Source .

Chapter	Threshold on the coarse side (step m)	Threshold on the fine side (step $m+1$)	Note
9-07, 11-00, 11-01, 11-08	t_m	t_{m+1}	The calendar's notation as is
11-02	—	t $(= t_N)$	The threshold of the finest step N
11-03, 11-04	$\tau = \mathrm{temp}(x)$	$t = \mathrm{fiery}(x, L)$	$x = \hat{\beta}_m$
11-05	—	$t = \mathrm{fiery}(x, L)$	$x = \hat{\beta}_m$
11-06, 11-07	t_m	t_y	y is the reference coupling of the fine step
11-09	$t = \hat{g}P$	—	$x = \hat{\beta}_m$, $\hat{g} = x^{-1/2}$. The opposite side from the t of 11-03

The t in § 02 and § 03 of this chapter is the threshold of the budget point of the representation in question (t_m for the representation at step m).

The **representation** `Rep` (`YM/RG/History.lean:165`) is a tuple of six ingredients Source : the path data (the paths entering the action and their coefficients), the truncation constants `Clip` (the "truncation parameters" of 9-06), the family of activities `Activities`, the budget point `BudgetPoint` (the "ledger point" of 9-06: threshold, covering scale, order of derivatives and so on), the coupling β, and the vacuum constant v. Its value is

$$\mathrm{eval}(V) = \exp\Big(v\,|\Lambda_m| - S_\beta^{\mathrm{ch}}(V) - \sum_i \mathbf{1}[\text{healthy}]\,E_i(V)\Big)\cdot \mathrm{gas}(V) \tag{1}$$

(`Rep.exponent`, `Rep.eval`, :176, :179) Source . S_β^{ch} is the path action averaged over the signed orientations of the hypercubic group (384 of them) (`PathData.inChannels`, `YM/RG/PathActions.lean:241`), E_i are the regular activities, and gas is the gas of forced activities.

The **induction hypothesis** `History.Conv N p m` (`YM/RG/History.lean:213`) has four fields Source .

1. The budget point is that of step m (threshold t_m, covering scale B_m and so on).
2. The truncation constants are the constants of the step.
3. The representation is `Lawful`: the path data satisfy `Bounds`, the activities are `Valid`, the ledger of regular activities is $\le \hat g_m^{11/5}$, and the ledger of forced activities is $\le e^{-P_m^{1/5}}$.
4. The coupling is in the band: $\beta_{N,p,m} \in [\hat\beta_m/2,\, 2\hat\beta_m]$.

The **one-step relation** `History.Transfer N p m` (:223) consists of two formulas Source .

$$\mathrm{eval}_m = \mathrm{push}_{\beta_{m+1}}\,\mathrm{eval}_{m+1}, \qquad \big|\beta_{m+1} - \beta_m - s\log b_m\big| \le \mathrm{drift}_{m+1}. \tag{2}$$

The push-forward $\mathrm{push}_\beta F(V) = \int T_\beta(U,V)F(U)\,dU$ is the integral against the one-step kernel (`YM/RG/Block.lean:77`), and the β of the kernel is the coupling of the step on the **fine side**. The second formula bounds the deviation from the one-loop running ($s = 11C_g/(24\pi^2)$ is the one-loop slope).

The **statement for all steps** `History.Selected` (:234) bundles the following for all N, p and $m \le N$ Source : the path data do not depend on the volume p (`path_shared`); the induction hypothesis at each step (`represents`); at step N, with $v = 0$, the

representation equals the bare Wilson weight exactly (`starts`, :237); the one-step relation at each step (`steps`); $\beta = \bar{\beta}$ at step 0 (`reaches`, :239); and the running (`running`).

As a corollary, the partition function is preserved through the steps Source (`History.partition_eq`, :242):

$$\int \mathrm{eval}_{N,p,m}\, dV = Z_{\Lambda_N}(\beta_{N,p,N}) \qquad (m \le N). \tag{3}$$

Which target it enters. The certificate of the construction's family, `Family.Makes` (`YM/Source/Construction.lean:44`), contains `path.Selected`, and the family appears in the conclusion of `constructed`. The objects of this chapter lie on the chain of the existence theorem Source .

02

The truncated action and the path action

For the holonomy U_p of a plaquette p we use the chord distance $\sigma = \mathrm{chord}(U_p) = \|U_p - 1\|/\sqrt{d}$ (d is the dimension of the representation; $\mathrm{price}(U_p) = 1 - \mathrm{Re}\,\mathrm{tr}\, U_p/d = \sigma^2/2$). The **truncated action** (`PathData.clipped`, `YM/RG/PathActions.lean:218`) is

$$\mathrm{clip}_t(\sigma) = \begin{cases} c_{\mathrm{dir}}\,\sigma^2/2 & (\sigma \le t) \\ c_{\mathrm{lin}}\, t\,\sigma & (\sigma > t) \end{cases}, \qquad 0 < c_{\mathrm{lin}} < c_{\mathrm{dir}}/16 \tag{4}$$

(`Clip`, :139; the last inequality is the field `kink`) Source . Above the threshold t, the action drops from quadratic to linear. At $\sigma = t$ the value jumps downward, and the function is neither monotone nor convex Source .

The basic term of a plaquette (`baseTerm`, :221) is $\mathrm{clip}_t(\sigma_p)$, plus the square of the transported curvature $\|\ell_p\|^2/2$ only when the field around the plaquette is small (`kept`, :210: the chords of all plaquettes are bounded by a threshold that loosens with distance from the plaquette). The path action (the "main action" of 9-06) is

$$S\beta(V) = \beta\Big(\sum_p \mathrm{base}_p(V) + \sum_x (\text{cubic and quartic atoms})\Big) + \sum_x (\text{one-loop atoms}) \tag{5}$$

(`principal`, :236), and S_β^{ch} is its average over orientations Source .

Meaning Inferred : by weakening the action to linear above the threshold and keeping it in the representation, the rarity of large-field configurations remains explicitly in the action terms rather than in the gas. Having weakened the action handed to the coarse step produces prices as a "positive remainder" in the one-step integral (11-04).

03

The two kinds of activity, and the gas

Flags and health. At threshold t, the set of points touching plaquettes whose chord exceeds t is called the flags $\mathrm{flags}_t(V)$ (`YM/RG/Masks.lean:34`). When a support (the "support" of 9-06) X does not meet the flags, X is healthy (`Healthy`, :41) Source .

A **regular activity** (`RegularAddress`, `E`) is a function E_i with an anchor point and a support, and it enters the density only when healthy (the indicator in `masked`, :70). The condition `RegularFamily` (`YM/RG/Activities.lean:70`) has three parts: smooth up to finite order where healthy, $E_i(1) = 0$ at the flat configuration, and log-Hessian 0 in the affine directions at the anchor Source . The last two are the shape after the marginal terms (constant and quadratic) have been moved into the vacuum term and the coupling (11-09).

A **forced activity** (`ForcedAddress`, `K`) is a function K_i with a support and a non-empty "inventory" inside it. If $K_i(V) \neq 0$, the intersection of the support with the flags equals the inventory exactly (`permit`) Source . The gas is

$$\mathrm{gas}(V) = \sum_b \mathbf{1}\Big[\text{supports pairwise disjoint},\ \bigcup_{i\in b} \mathrm{inv}_i = \mathrm{flags}_t(V)\Big] \prod_{i\in b} K_i(V) \tag{6}$$

(`admissible`, `gas`, `YM/RG/Activities.lean:114-129`) Source . If the flags are empty, the gas is 1 (`gas_one`, :135). In other words, forced activities **live only on the flags of the field at that step**.

Ledgers. Both kinds of activity carry a ledger in the form of a pointwise sum (`EBound`, `KBound`, :98, :102) Source :

$$\sup_x \sum_{i:\,\mathrm{site}_i = x} e^{c\sup \mathrm{cov} B(X_i)}\, \|E_i\|_{t,k} \le \hat{g}_m^{11/5}, \qquad \sup_x \sum_{i:\,x\in X_i} e^{c\sup \mathrm{cov} B(X_i)}\, \sup|K_i| \le e^{-P_m^{1/5}}. \tag{7}$$

The sum for regular activities is over pieces anchored at x; the sum for forced activities is over pieces whose support contains x. c_{sup} is the covering exponent of the budget point (`supportExponent`). $\mathrm{cov}_B(X)$ is the smallest number of balls of radius B in the lattice maximum distance (cubes of side $2B+1$ with a centre), arranged so that their union is connected, needed to cover the support (`coverNumber`, `YM/Lattice/Support.lean:71`), and $\|E\|_{t,k}$ is the supremum over healthy configurations of derivatives up to order k in directions of size t (`supBounds`, `YM/RG/Masks.lean:164`) Source . Because the sums are pointwise, the ledgers themselves do not depend on the volume Source .

04

What this book's study had seen

This book's study also held the large-field corrections as "activities per hole", and assumed estimates of the form $|z_t(\gamma)| \le \exp(-\gamma_E \beta S^\flat(\gamma))$ at each step and each hole (the "large-field estimate" and "large-field gas" in the premise list, 28-04). There are two differences.

- This book's premises put the rarity $e^{-c\beta}$ on the side of the activities, and stacked up regions step by step. The published construction keeps the rarity on the side of the truncated action, and the ledger of activities is the weak $e^{-P^{1/5}}$ Inferred .
- For this book's deterministic block map, it has not been shown that the coarse density keeps a representation of the form (1) at every step. Carrying the representation itself through all steps is the component this book lacks Inferred (11-10 § 06).

05

What has not been read

- The meaning of all the fields of `Bounds` for the path data (`YM/RG/PathActions.lean:173`). We have read only as far as the names `metric_zero` and `loop_jets`.
- The role of `extraTest` (an additional condition) for forced activities.
- How the difference between the orientation average S^{ch} and the path action of a single orientation is handled in the accounting of one-step prices (11-07 § 06).

What this chapter can say

Can say	Cannot say
Source The density at each step has a representation of the form (1), and the induction hypothesis requires the ledgers (7) and the coupling band	That the representation is unique (the source states only existence)
Source Steps are linked by the push-forward identity (2), and the partition function is preserved (3)	Reflection positivity of the push-forward measure (not stated)
Source The truncated action is linear above the threshold, with linear coefficient less than $c_{\mathrm{dir}}/16$	That the truncated action is "an approximation of the effective action" (it is only a component of the representation)
Source Forced activities live only on the flags, and the gas is 1 when the flags are empty	That the gas is bounded uniformly in the volume (estimates at the level of the density include an exponential in the volume)

Sources and reproduction

Item	Kind	Source / tool
Representation, value, Lawful	Source	`YM/RG/History.lean:165` (`Rep`), :179 (`eval`), :181 (`Lawful`)
Induction hypothesis, one-step relation, all steps, partition function	Source	`YM/RG/History.lean:213`, :223, :234, :242
Truncated action	Source	`YM/RG/PathActions.lean:139` (`Clip`), :210 (`kept`), :218 (`clipped`), :221 (`baseTerm`), :241 (`inChannels`)
Activities, gas, ledgers	Source	`YM/RG/Activities.lean:62` (`Activities`), :70, :98, :102, :106 (`Valid`), :129 (`gas`), :135 (`gas_one`)
Flags, health, ledger weights	Source	`YM/RG/Masks.lean:34`, :41, :70, :144 (`mass`), :164 (`supBounds`)
Certificate of the family	Source	`YM/Source/Construction.lean:44` (`Family.Makes`)

Words used in this chapter — representation, truncated action, flags, healthy, regular activity, forced activity, gas, ledger, covering number. Their meanings are in the A-02 glossary.

Next chapter to read: 11-02 The finest step — the first move, splitting the bare Wilson weight into this representation.

The finest step — splitting the bare Wilson weight into the representation

> This chapter is established knowledge (a reading of the published construction). § 04 is this book's own investigation and § 05 is a map of open questions.

What this chapter shows — The induction starts at step N from "the value of the representation equals the bare Wilson weight exactly". We show the plaquette-by-plaquette identity that builds that representation, and that the remaining difference, due, is non-negative and at least $t^2/4$ on large plaquettes. This is why the first forced activities are small.

Chapters assumed — 11-01 The form of the density at each step (truncated action, flags, health, the two kinds of activity).

Said first — The identity is an equality per plaquette, with no factor of β. How the first activities are arranged into polymers and entered into the ledgers, this book has read only as far as names and docstrings. The lemmas of this chapter are reached in the verification record from one of the headline theorems, but we have not checked whether they lie on the chain of the existence theorem.

The order of this chapter

1. Statement — the starting condition and the plaquette-by-plaquette identity
2. Outline of the mechanism — splitting into four terms
3. Where the difference goes — the first regular activities and forced activities
4. What this book's study had seen
5. What has not been read

Labels: Source = read in the source of the published construction / Inferred = this book's inference / Assessment = according to the committee's assessment. The other labels (Proved here, Cited, Reading, Lean and so on) mean the same as in the rest of the book (for details see "Labels" in 6-00).

01

Statement — the starting condition and the plaquette-by-plaquette identity

Starting condition. The field `starts` of the statement for all steps `History.Selected` (`YM/RG/History.lean:237`) requires that the representation at step N has $v = 0$ and that its value equals the bare Wilson weight exactly Source :

$$\mathrm{eval}_{N,p,N}(U) = \exp\Big(-\beta_N \sum_{\square} \mathrm{price}(U_{\square})\Big), \qquad \mathrm{price}(u) = 1 - \tfrac{1}{d}\mathrm{Re}\,\mathrm{tr}\,u = \tfrac{1}{2}\mathrm{chord}(u)^2. \tag{1}$$

Here the subscript p of eval is the volume index, and the sum is over all plaquettes $\square$. The volume index does not appear from equation (2) on, so there we write a plaquette as p. The starting field on the side of the one-step theorem, `Recipe.Starter` (`YM/Shoot/Criteria.lean:93`), requires the same two things ($v = 0$, value equal to the bare weight) Source .

The plaquette-by-plaquette identity `WilsonStartup.due_eq` (`YM/Extra/WILSON_STARTUP/Ownership.lean:68`) Source : if the truncation constant satisfies $c_{\mathrm{dir}} \le 1$, then for every configuration U and plaquette p

$$\mathrm{price}(U_p) = \mathrm{base}_p(U) + \mathbf{1}[\mathrm{healthy}]\, Q_p(U) + (1 - c_{\mathrm{dir}})\, \mathbf{1}[\mathrm{healthy}]\, R_p(U) + \mathrm{due}_p(U). \tag{2}$$

Here base_p is the basic term of the first step's path data `top` (`Design.lean:34`) (the truncated action, plus the quadratic term when `kept`), Q_p is the quartic term in the curvature (coefficient $1 - c_{\mathrm{dir}}$), R_p is the remainder of the price after removing the quadratic and quartic parts (`surplus`, `Curvature.lean:166`), and "healthy" means that the points the plaquette touches are not in the flags. Three things are stated about the difference due_p Source .

- `due_range`: $0 \le \mathrm{due}_p \le 2$ (under $c_{\mathrm{dir}} \le 1$).

- `due_large`: on plaquettes whose chord exceeds the threshold ($\mathrm{chord}(U_p) > t$), $\mathrm{due}_p \geq t^2/4$.
- `owed_zero` (:193): the same quantity read in the orientation frame is 0 outside the B-halo of the flags. The hypothesis is the whole of `Decent` ($c_{\mathrm{dir}} \leq 1$, $B \geq 2$, and conditions on the threshold and the loosening length).

Which target it enters. `due_eq` and `owed_zero` are "used" in the verification record, and the witness in the record is the headline theorem `SharpGram.habitat`. We have not checked whether they lie on the chain of the existence theorem Source .

02

Outline of the mechanism — splitting into four terms

Let the chord be $\sigma = \mathrm{chord}(U_p)$ and the threshold t. The first path data `top` take the plaquette itself as the path, with transported curvature $\ell_p = \sqrt{1 - c_{\mathrm{dir}}}\,\mathrm{curv}(U_p)$ (`ell_sq`, `Design.lean:86`: $\|\ell_p\|^2 = (1 - c_{\mathrm{dir}})\|\mathrm{curv}\|^2$) Source . The definition of due (`due` in `Ownership.lean`) splits into three cases Source .

Case	Part entered into the representation	due
$\sigma > t$ (large plaquette)	Only the truncated action $c_{\mathrm{lin}} t\sigma$	price $- c_{\mathrm{lin}} t\sigma$
$\sigma \leq t$, healthy and `kept`	$c_{\mathrm{dir}}\sigma^2/2 + \|\ell\|^2/2 +$ quartic + remainder	0
$\sigma \leq t$, otherwise	$c_{\mathrm{dir}}\sigma^2/2$ (and, if healthy, quartic and remainder)	The surplus or shortfall of the quadratic term (within $[0, 2]$)

For small fields (second row), c_{dir} times the bare price $\sigma^2/2$ enters the representation as the quadratic part of the truncated action, and the remaining $1 - c_{\mathrm{dir}}$ times enters as the squared curvature, the quartic term and the remainder; no difference is left Source (formula).

The **lower bound on large plaquettes** is a short computation (the line of the proof of `due_large`) Source . From the condition on the truncation constants $c_{\mathrm{lin}} < c_{\mathrm{dir}}/16 \leq 1/16$ we get $c_{\mathrm{lin}} \leq 1/4$. Since $t < \sigma$,

$$\mathrm{due}_p = \tfrac{1}{2}\sigma^2 - c_{\mathrm{lin}}\, t\,\sigma \;\geq\; \tfrac{1}{2}\sigma^2 - \tfrac{1}{4}\sigma^2 = \tfrac{1}{4}\sigma^2 > \tfrac{1}{4}t^2. \quad (3)$$

In other words, on a plaquette above the threshold the truncated action puts at most half of the bare price into the representation, and sends at least $t^2/4$ of the rest out as

due.

03

Where the difference goes — the first regular activities and forced activities

Multiplying identity (2) by β_N, summing and exponentiating, the bare weight becomes the product "exponential of the truncated action $\times$ exponential of the terms where healthy $\times$ $e^{-\beta_N \sum \text{due}}$". The headline docstring in the source says: "all the remaining (non-negative) difference is charged to the forced packets; there is no logarithmic addition in this plan" Source (docstring).

- We read the remainder terms where healthy as becoming the first regular activities (location `YM/Extra/WILSON_STARTUP/Ink.lean`) Inferred .
- We read due as becoming the first forced activities, on polymers formed by joining overlapping B-balls of the flags (location `YM/Extra/WILSON_STARTUP/Packets.lean`) Inferred .

Size Inferred : from the definition of the threshold, $\hat{\beta}_N t_N^2 = P_N^2$. The actual coupling β_N is in the band $[\hat{\beta}_N/2,\, 2\hat{\beta}_N]$ (`Conv.band`, `YM/RG/History.lean:218`), so $\beta_N t_N^2 \geq P_N^2/2$, and per flagged plaquette $e^{-\beta_N \text{due}} \leq e^{-P_N^2/8}$. The first forced activities start much smaller than the induction-hypothesis ledger $e^{-P_N^{1/5}}$.

04

What this book's study had seen

The ultraviolet chain of this book's study set step N as the bare Wilson measure and stacked up large-field regions from the first step (the premise "large-field estimate" in 28-04). It did not have this way of building the initial condition: truncating the action at the threshold at the first step and pushing the difference onto activities as a bounded factor. We note it as something worth comparing.

05

What has not been read

- The construction that arranges the first forced activities into polymers (`YM/Extra/WILSON_STARTUP/Packets.lean`), and their entry into the first ledger (`Treats.start`, location unconfirmed).
- The part that turns the remainder where healthy into the first regular activities (`YM/Extra/WILSON_STARTUP/Ink.lean`), and its ledger $\hat{g}_N^{11/5}$.

- Whether `due_eq` lies on the chain of the existence theorem (the witness in the record is `SharpGram.habitat`).
- The details of the correspondence between `owed` (due read in the orientation frame) and `due`.

What this chapter can say

Can say	Cannot say
Source The representation at step N has $v = 0$ and equals the bare Wilson weight (`Selected.starts`)	That this representation is unique
Source The plaquette-by-plaquette identity (2): bare price = basic term + quartic and remainder where healthy + due ($c_{\mathrm{dir}} \le 1$)	The assembly from the identity to the first ledgers (not read)
Source $0 \le \mathrm{due} \le 2$, and $\mathrm{due} \ge t^2/4$ on large plaquettes	That due is large on every plaquette near the flags (it can be 0 on small plaquettes)
Inferred The first forced activities are of order at most $e^{-P_N^2/8}$ per flag	That this size is written as a statement (this book has not seen it)

Sources and reproduction

Item	Kind	Source / tool
Starting condition	Source	`YM/RG/History.lean:237` (`Selected.starts`), `YM/Shoot/Criteria.lean:93` (`Starter`)
The plaquette-by-plaquette identity and its properties	Source	`YM/Extra/WILSON_STARTUP/Ownership.lean:68` (`due_eq`), `due_large` and `due_range` (same file), :193 (`owed_zero`)
The first path data	Source	`YM/Extra/WILSON_STARTUP/Design.lean:23` (`bulk`), :34 (`top`), :86 (`ell_sq`), `Curvature.lean:166` (`surplus`)
Dependency record	Source	`verification/declaration-usage.tsv` (`due_eq` and `owed_zero` are used, witness `SharpGram.habitat`)

Words used in this chapter — bare Wilson weight, truncated action, flags, healthy, due (the remaining difference), forced activity, regular activity. Their meanings are in the A-02 glossary.

Next chapter to read: 11-03 Decomposing the one-step action, and regions — inside the one-step integral, the action is split into local rows and configurations are sorted by seed type.

Decomposing the one-step action, and regions

> This chapter is established knowledge (a reading of the published construction). § 06 is this book's own investigation and § 07 is a map of open questions.

What this chapter shows — The algebraic identity that writes the "fine action + penalty − coarse action" appearing in the one-step integral as a sum of local rows, one per plaquette and per edge, and the regions that sort configurations by three sets of seeds (the seed type). The regions overlap and are glued with a smooth partition of unity. The region conditions are imposed globally, and the estimates use a form localised around the seeds.

Chapters assumed — 11-01 (truncated action, flags), 9-01 The block kernel (predictor and penalty), 9-04 The predictor and projection to the group.

Said first — The row identity is an algebraic equality and contains no estimate. The estimate (energy per seed) is 11-04. The declarations of this chapter are reached in the verification record from one of the headline theorems, but we have not checked whether they lie on the chain of the existence theorem.

The order of this chapter

1. Setting — the one-step integral and three thresholds
2. Statement 1 — the row identity
3. Statement 2 — seed types and regions
4. Statement 3 — the partition of unity, and flags always touch a seed
5. Outline of the mechanism — why overlapping regions, why both global and local
6. What this book's study had seen
7. What has not been read

Labels: Source = read in the source of the published construction / Inferred = this book's inference / Assessment = according to the committee's assessment. The other labels (Proved here, Cited, Reading, Lean and so on) mean the same as in the rest of the book (for details see "Labels" in 6-00).

01

Setting — the one-step integral and three thresholds

Let the one-step data be S (from the fine lattice f to the coarse lattice c, block side L). There is a fine configuration U, a coarse configuration V, and a discrete choice inside the kernel (gauge frame and predictor label) k; the predictor $Q = Q(U, k)$ is the value on coarse edges built from the fine field (9-04). We write the "deviation" of a coarse edge b as $\mathrm{wire}_b = \mathrm{chord}(V_b Q_b^{-1})$ (`YM/Pressure/Cost.lean:187`) Source .

Let the reference coupling of the coarse step be x ($= \hat{\beta}_m$); we use three quantities (`YM/Pressure/Seeds.lean:22-27`) Source .

Symbol	Definition	Meaning
$\tau = \mathrm{temp}(x)$	$x^{-1/2}\exp((\log\log x)^2)$	Threshold of the coarse step
$t = \mathrm{fiery}(x, L)$	$\mathrm{temp}(x + s\log L)$ (s is the one-loop slope)	Threshold of the fine step
$R = \mathrm{radiusB}(x)$	$\lceil(\log x)^2\rceil$	Scale of the halo (the neighbourhood around a seed)

When $x = \hat{\beta}_m$, in the notation of 9-07 and 11-01 we have $\tau = t_m$ (coarse side), $t = t_{m+1}$ (fine side), and $R = B_m$ (covering scale) (correspondence of threshold letters in 11-01).

The "size" of a plaquette is $\mathrm{toll}(U_p) = \mathrm{chord}(U_p)$ (`toll` in `YM/Pressure/Rows.lean:55`).

02

Statement 1 — the row identity

Write the action of a fine plaquette as crown_p^f (truncated action + the quadratic term when `kept`), the action of a coarse plaquette as crown_q^c, and the penalty term of a coarse edge as pen_b (`monetary`, `Rows.lean:112`). The **row identity** `Tube.locates` (`YM/Pressure/Rows.lean:145`) is Source

$$\sum_{f} \mathrm{workF} + \sum_{b} \mathrm{workI} + \sum_{q} \mathrm{workC} + \sum \mathrm{workN} + \sum_{q} (\mathrm{workL} + \mathrm{workD}) = \sum_{p} \mathrm{crown}_{p}^{f} + \sum_{b} \mathrm{pen}_{b} - \sum_{q} \mathrm{c}$$

The proof is only a sum of linear identities (`linarith`). The contents of the rows are as follows[Source] . Let X_q^c be the curvature of the coarse plaquette q multiplied by $\sqrt{c_{\mathrm{dir}}}$, and shrink it by the threshold τ:

$$\mathrm{vision}_q = \min\bigl(1,\ \tau / \mathrm{toll}(V_q)\bigr)\, X_q^c \quad (2)$$

(`thinned`, :71, `vision`, :104); distributing this to the fine plaquettes with the blocking kernel Φ (`leap`, :46) gives $M_v = \Phi \cdot \mathrm{vision}$.

- $\mathrm{workC}_q = \langle \mathrm{vision}_q, X_q^c \rangle - \frac{1}{2}\|\mathrm{vision}_q\|^2 - \mathrm{crown}_q^c$ (the coarse-plaquette row, :129).
- $\mathrm{workF}_p = \mathrm{crown}_p^f + \frac{1}{2}\|M_v\|^2 - \langle M_v, X_p^f \rangle$ (the fine-plaquette row, :122).
- workI_b: completing the square of the penalty term with the vision distributed to the noise side (:126).
- workL_q: the inner product of vision with the "deviation" between the coarse curvature and the curvature rebuilt from the fine curvature and the noise (:133). workN and workD are the remainders needed to close the squares.

Meaning[Inferred] : it compares the coarse curvature with "the blocked fine curvature + noise", completes the square, and distributes the difference into local rows. Each row depends only on the neighbourhood of a single plaquette or edge.

03

Statement 2 — seed types and regions

A **seed type** `Stores` (`YM/Pressure/Seeds.lean:34`) is a triple of finite sets $A = (\mathrm{costs}, \mathrm{aisles}, \mathrm{flags})$[Source] : the set of large fine plaquettes, the set of coarse edges that deviate greatly from the prediction, and the set of flag points of the coarse step. The **number of seeds** is $\#A = |\mathrm{costs}| + |\mathrm{aisles}| + |\mathrm{flags}|$ (`shares`, :46). Here a "seed" is a working term of the published construction meaning each individual element of these three sets.

The **global region condition** `Seeded A U k V x W` (`YM/Push/SECTOR_MEASURE/Cells.lean:51`)[Source] imposes the following on **every** fine plaquette p and every coarse edge b.

$$\begin{aligned} &p \in \text{costs} \Rightarrow \text{toll}(U_p) \geq t/4, \qquad p \in/\ \text{costs} \Rightarrow \text{toll}(U_p) \leq t/2, \\ &b \in \text{aisles} \Rightarrow \text{wire}_b \geq Wt, \qquad b \in/\ \text{aisles} \Rightarrow \text{wire}_b \leq 2Wt, \\ &\text{flags} = \text{flags}_\tau(V) \quad \text{(exactly equal)}. \end{aligned} \tag{3}$$

W is a constant taken large. The plaquette and edge conditions have a margin between $t/4$ and $t/2$ (between Wt and $2Wt$), that is, hysteresis. The flags have no margin and are made to coincide exactly with the flags of the coarse field.

The **local region condition** `Plot A U k V x W` (`YM/Pressure/Seeds.lean:51`) Source imposes the same conditions only on plaquettes and edges touching the radius-$5R$ neighbourhood of where the seeds sit (`dome (5*radiusB x)`), requires the flags to coincide there, and adds that the frame weight of the choice k is positive. The local condition follows from the global one (`Seeded.plot`, `Cells.lean`: if the likelihood of the choice k is positive) Source .

The object of the estimate is the sum of rows over the radius-$2R$ neighbourhood of the seeds,

$$\text{assessed}(A) = \sum_{\text{dome}(2R)} \big(\text{workF} + \text{workI} + \text{workC} + \text{workN} + \text{workL} + \text{workD}\big) \tag{4}$$

(`Stores.assessed`, `Seeds.lean:85`) Source .

04

Statement 3 — the partition of unity, and flags always touch a seed

The **partition of unity** `Allocation` (`YM/Push/SECTOR_MEASURE/Profile.lean:30`) consists of weights $\text{bill}_A(k, U, V)$, one per seed type, that satisfy Source

$$\text{bill}_A \geq 0, \qquad \sum_A \text{bill}_A = 1, \qquad \text{bill}_A \ /= 0 \ \Rightarrow\ \texttt{Seeded}\ A. \tag{5}$$

and are measurable. Wherever a weight is not 0, the **global condition** holds, and if the choice is live the local condition `Plot` can be extracted (`plots`, :50) Source .

Flags always touch a seed (`flagged`, :56) Source : if bill$_A$ $/= 0$ and a point i is in the flags at the fine-step threshold t, then there is a plaquette $p \in$ costs touching i. The proof: a flagged plaquette has toll $> t > t/2$, so by the contrapositive of the second part of the global condition (3) it is in costs. Since forced activities coming in from the previous step live only on the flags of the fine step (11-01 § 03), **the inventory points of incoming activities always touch a seed of this step (a plaquette in costs)** Source (statement read) / Inferred (reading of its meaning).

Which partition is used. The version that splits by indicator functions, `Allocation.hard` (:68; `booking`, `Cells.lean:87`, cuts exactly at thresholds $t/4$ and Wt), is outside the dependencies Source (record). What is "used" in the record is the family of product weights built from a smooth transition function χ,

$$\mathrm{share}_J = \prod_{i \in J} (1 - \chi((r_i/d)^2)) \prod_{i \in /J} \chi((r_i/d)^2), \qquad \sum_J \mathrm{share}_J = 1 \quad (6)$$

(`Chi.share`, `share_sum`, `Profile.lean:126-133`) Source . χ is a smooth transition equal to 1 for $y \le 1$ and 0 for $y \ge 4$; if $\chi((r/d)^2)$ $/= 0$ then $r < 2d$, and if $1 - \chi((r/d)^2)$ $/= 0$ then $r > d$ (`chosen_lt`, `exterior_gt`), so setting $d = t/4$ matches the margin $[t/4, t/2]$ of (3) Inferred (how d is assigned has not been read).

05

Outline of the mechanism — why overlapping regions, why both global and local

1. **Overlapping regions.** Cutting exactly at a threshold makes the region change discontinuously for configurations near the threshold. Regions with a margin overlap, and gluing them with a smooth partition of unity avoids boundary problems when regions are differentiated or moved inside the integral Inferred . The source's docstring also says that the indicator-function split is for a measurable partition and is not used as a smooth weight Source (docstring).
2. **Both global and local.** The condition `Seeded` attached to the partition of unity is a condition imposed on the whole configuration. The estimate (energy per seed), on the other hand, uses `Plot`, localised to the $5R$ neighbourhood of the seeds, and the sum of rows (4) over the radius-$2R$ neighbourhood. Because the localised form is extracted from the global condition, the estimate side need only look near the seeds Source (read `plots`) / Inferred (reading of the intent).

3. **Scale of the halo.** $R = \lceil (\log x)^2 \rceil$ is measured by the $\hat{\beta}$ of the coarse step. The local condition uses $5R$ and the estimated sum $2R$ Source .
4. **The integral with one region fixed** is the unit of the large-field analysis of one step. A region is determined by the field at that step alone and is not inherited from the previous step Source (read the fields of `Stores`) / Inferred (reading of its meaning).

06

What this book's study had seen

This book's study sorted configurations by small-field indicator functions and thought in terms of stacking up the large-field region Ω^c step by step and fixing it thereafter. The integral with one region A fixed corresponds to this book's "integral with Ω^c fixed" Inferred . There are two differences.

- The regions of the published construction **are determined by that step alone** and keep no history of regions. The locations of incoming activities are automatically included in this step's seeds by `flagged`.
- The published construction uses overlapping regions with margins and a smooth partition of unity. This book's study cut with indicator functions.

A formula corresponding to the row identity (1), writing the difference of one-step actions by local completion of squares, did not exist on this book's side.

07

What has not been read

- The assembly of the partition of unity actually used on the main-theorem side (where the `Chi` weights are assembled into an `Allocation` over seed types), and how d is assigned.
- The content of the description of overlaps between regions (`YM/Push/SECTOR_MEASURE/Overlap.lean`).
- How the blocking kernel Φ (`leap`) and the distribution to the noise Φ' (`kick`) are built, and the details of the meaning of rows workN and workD.
- The role of the condition that the frame weight of the choice k is positive (field `list` of `Plot`).

What this chapter can say

Can say	Cannot say
Source Fine action + penalty − coarse action equals a sum of local rows (algebraic identity (1))	That any of the rows is positive (that is the estimate of 11-04)
Source The region condition `Seeded` is imposed globally, and the local `Plot` (radius $5R$) is extracted afterwards	That the region condition says anything about the field outside the halo (`Plot` does not)
Source Hysteresis applies only to costs and aisles; flags are made to coincide exactly with the flags of the coarse field	That the regions form a partition of configurations (they overlap)
Source If the weight of the partition of unity is not 0, every fine flag point touches a plaquette in costs (`flagged`)	That this lemma lies on the chain of the existence theorem (the witness in the record is `SharpGram.habitat`)
Source The indicator-function partition `hard` is outside the dependencies; the smooth `Chi.share_sum` is used	The details of assembling the smooth partition

Sources and reproduction

Item	Kind	Source / tool
Thresholds and halo	Source	`YM/Pressure/Seeds.lean:22` (`radiusB`), :25 (`temp`), :26 (`fiery`)
Deviation	Source	`YM/Pressure/Cost.lean:187` (`wire`)
Rows and the identity	Source	`YM/Pressure/Rows.lean:46` (`leap`), :71 (`thinned`), :104 (`vision`), :112 (`monetary`), :122, :126, :129, :133 (rows), :145 (`locates`)
Seed types, local regions, estimated sum	Source	`YM/Pressure/Seeds.lean:34` (`Stores`), :46 (`shares`), :51 (`Plot`), :85 (`assessed`)
Global regions, indicator-function split	Source	`YM/Push/SECTOR_MEASURE/Cells.lean:51` (`Seeded`), :87 (`booking`)
Partition of unity	Source	`YM/Push/SECTOR_MEASURE/Profile.lean:30` (`Allocation`), :50 (`plots`), :56 (`flagged`), :68 (`hard`, outside), :85 (`Chi`), :133 (`share_sum`)

Words used in this chapter — row identity, seed, seed type, region, hysteresis, partition of unity, flags, halo. Their meanings are in the A-02 glossary.

Next chapter to read: 11-04 Energy per seed — three inequalities showing that, on a region, the sum of rows in the neighbourhood is positive in proportion to the number of seeds.

Energy per seed

> This chapter is established knowledge (a reading of the published construction). § 06 is this book's own investigation and § 07 is a map of open questions.

What this chapter shows — The exact form (order of quantifiers and hypotheses) of the one-step, configuration-by-configuration inequality $c_E\, t^2 \cdot \#\mathrm{seeds} \le \mathrm{assessed}$ on a region, and the three inequalities that build it. Truncating the action handed to the coarse step to linear produces a positive remainder, and the shrinking of linear quantities by L^{-2} under blocking lets the fine side win.

Chapters assumed — 11-03 (row identity, seed types, regions, the three thresholds), 11-01 § 02 (truncated action).

Said first — The inequality of this chapter is a one-step, configuration-by-configuration inequality, not a claim across steps. This book has not read the proofs of the three inequalities (location `YM/Work/E05` – `E07`); only the statements were read. The declarations of this chapter are reached in the verification record from one of the headline theorems, but we have not checked whether they lie on the chain of the existence theorem.

The order of this chapter

1. Statement — energy per seed (order of quantifiers and hypotheses)
2. Lower bound for coarse plaquettes — the positive remainder
3. Lower bound for fine plaquettes and the penalty
4. Payment for large coarse plaquettes
5. Assembly, and the factor that comes out of the integral
6. What this book's study had seen
7. What has not been read

Labels: Source = read in the source of the published construction / Inferred = this book's inference / Assessment = according to the committee's assessment. The other labels (Proved here, Cited, Reading, Lean and so on) mean the same as in the rest of the book (for details see "Labels" in 6-00).

01

Statement — energy per seed (order of quantifiers and hypotheses)

Notation follows 11-03 § 01: the reference coupling of the coarse step x, the coarse threshold $\tau = \mathrm{temp}(x)$, the fine threshold $t = \mathrm{fiery}(x, L)$, the halo scale $R = \mathrm{radiusB}(x)$, the seed type A, the number of seeds $\#A$, and the sum of rows in the neighbourhood $\mathrm{assessed}(A)$. The coarse threshold τ and the fine threshold t are t_m and t_{m+1} in the notation of 9-07 (correspondence of threshold letters in 11-01).

The **order bundle** `Teller` (`YM/Pressure/Seeds.lean:89`) is an input bundling the choice of kernel, the step constants of the fine and coarse steps, the number j of convolutions of the block weights, and so on. That the one-step data S and the tube T (the pair of path data for the fine and coarse steps) fit it is called `Funds` (the choice of kernel agrees, $J = j$, the kernel's three obligations `Branches`, `Bounds` and `Roll` for the tube's path data, and so on) Source .

The order of quantifiers in **energy per seed** `Teller.Grants` (`YM/Pressure/Seeds.lean:129`) Source is as follows (`Funds`, `live`, `Serves`, `cash`, :101-127).

$$\exists i\, \forall L \geq i\ \ \forall^{\infty} W\ \ \exists c_E > 0\ \ \forall^{\infty} x\ \ \forall (S, T, F, A, U, k, V) :\ \ c_E\, t^2\, \#A\ \leq\ \mathrm{assessed}(A). \quad (1)$$

$\forall^{\infty}$ means "for all sufficiently large". The range of the last $\forall$ carries all of the following hypotheses Source .

1. The block side of S is exactly L (`S.patch.L = L`).
2. **Lower bound on the lattice side**: side of the coarse lattice $\geq 100(1 + R)$.
3. S, T fit the order (`Funds`).
4. **Gauge section**: the fine field U is 1 on the trunk of the block tree (`SliceSet F U`, :48). It is an inequality on the tree gauge section.
5. The local region condition `Plot A U k V x W` (11-03 § 03).

c_E (in the source's wording κ in `Grants`, μ in the price fields) may depend on L and W, and the lower bound on x depends on L, W, c_E Source . The threshold t is the threshold on the **fine side**.

This statement holds for every order without hypotheses: `Next.Fluctuations.grants` (`YM/Next/FLUCTUATIONS/Grants.lean:24`) Source . Its content is `Integrated.pressure_grants` (`YM/Integrated.lean:222`), which assembles it with `Bid.issues` (:155) from ten component statements (E01–E09 in `YM/Bid/Energy.lean`, and the smallness outside, `EBare`). The components are proved under `YM/Work/`, and `EBare` is closed by `bare` in `YM/Next/FLUCTUATIONS/Bare.lean:64` Source .

02

Lower bound for coarse plaquettes — the positive remainder

Lower bound for coarse plaquettes: the structure `Bid.Crowds` (`YM/Bid/Energy.lean:94`) and `def E05` (:101), which requires it for every order Source : $\exists \delta > 0\ \exists B > 0\ \forall L\ \forall W$, for sufficiently large x, for configurations satisfying the region (with the hypotheses of § 01) and for every coarse plaquette q, setting $\sigma = \mathrm{toll}(V_q)$,

$$\begin{aligned} \tau < \sigma \le \delta \ &\Rightarrow\ \tfrac{c_{\mathrm{dir}}}{4}\,\tau\sigma \ \le\ \mathrm{workC}_q, \\ \sigma \le \tau \ &\Rightarrow\ -B\tau^4 \ \le\ \mathrm{workC}_q, \\ \sigma > \delta \ &\Rightarrow\ -c_{\mathrm{lin}}\,\tau\sigma \ \le\ \mathrm{workC}_q. \end{aligned} \tag{2}$$

All the thresholds are on the **coarse side** τ, and so are the truncation constants.

Meaning Inferred : on a medium plaquette ($\tau < \sigma \le \delta$), taking $|\mathrm{curv}| \approx \sigma$, vision is shrunk by the factor τ/σ, so the first two terms of the coarse-plaquette row are

$$\langle \mathrm{vision}, X^c \rangle - \tfrac{1}{2}\|\mathrm{vision}\|^2 \approx c_{\mathrm{dir}}\left(\tau\sigma - \tfrac{1}{2}\tau^2\right) \ \ge\ \tfrac{1}{2}c_{\mathrm{dir}}\,\tau\sigma. \tag{3}$$

On the other hand, the coarse-step action is the truncated action $c_{\mathrm{lin}}\tau\sigma$ ($c_{\mathrm{lin}} < c_{\mathrm{dir}}/16$). The difference is a positive remainder of at least $c_{\mathrm{dir}}\tau\sigma/4$. If the coarse-step action had stayed quadratic, $c_{\mathrm{dir}}\sigma^2/2$, the same computation would give $\mathrm{workC} \approx -c_{\mathrm{dir}}(\sigma-\tau)^2/2 \le 0$. **The positive remainder comes from truncating the action handed to the coarse step to linear**.

03

Lower bound for fine plaquettes and the penalty

Lower bound for fine plaquettes and the penalty: `Bid.Seals` (`YM/Bid/Energy.lean:103`; `def E06` is :110) Source : $\exists N\ \forall L \geq N\ \forall^{\infty} W\ \exists d > 0$, for sufficiently large x, under the hypotheses of § 01,

$$\begin{aligned} \tfrac{c_{\mathrm{lin}}^{I}}{2}\, t \sum_{p \in \mathrm{costs},\ \mathrm{toll} > t} \mathrm{toll}(U_p) \;+\; d\, t^{2}\, \#\{p \in \mathrm{costs} : \mathrm{toll} \leq t\} \;\leq\; \sum_{\mathrm{dome}(2R)} \mathrm{workF}, \\ \tfrac{c_{\mathrm{pen}}}{8} \sum_{b \in \mathrm{aisles}} \mathrm{wire}_b^{2} \;\leq\; \sum_{\mathrm{dome}(2R)} \mathrm{workI}. \end{aligned} \tag{4}$$

The threshold is on the **fine side** t, and c_{pen} is the kernel's penalty coefficient (in the source's wording `S.kernel.penalty`).

Meaning Inferred : the cross term of the fine-plaquette row $\mathrm{workF} = \mathrm{crown}^{f} + \tfrac{1}{2}\|M_v\|^{2} - \langle M_v, X^{f} \rangle$ is of order $(C/L^{2})\, t\,\mathrm{toll}$, by the upper bound on size (`Maximum`, :83: $\|M_v\| \leq (C/L^{2})\, t$). $L \geq N$ is needed to make this at most half of the linear term of the truncated action, $c_{\mathrm{lin}} t\,\mathrm{toll}$.

04

Payment for large coarse plaquettes

The third line of the coarse-plaquette lower bound can be negative, particularly on very large coarse plaquettes ($\sigma > \delta$). What pays for its total is the **payment for large coarse plaquettes** `Bid.Lapse` (`YM/Bid/Energy.lean:121`; `def E07` is :130) Source : $\exists C > 0\ \forall L\ \forall W > 0\ \exists Z > 0$, for sufficiently large x, under the hypotheses of § 01,

$$\sum_{q \in \mathrm{hit}} (\mathrm{workL} + \mathrm{workD})_q + \frac{C}{L^{2}}\, t \sum_{\substack{p \in \mathrm{costs} \\ \mathrm{toll} > t}} \mathrm{toll}(U_p) + C\, t \sum_{b \in \mathrm{aisles}} \mathrm{wire}_b + Z\, t^{2} R^{4}\, \#A_{\mathrm{sp}} \;\geq\; c_{\mathrm{lin}}^{\mathrm{c}}\, \tau \sum_{\substack{q \\ \mathrm{toll}(V_q) > \delta}} \mathrm{to}$$

The threshold on the left-hand side is the fine-side t, and on the right-hand side the coarse-side τ. $\#A_{\mathrm{sp}}$ is the number of seeds made only from especially large plaquettes (of order $\mathrm{toll} > \beta_{f}^{-0.49}$) and deviations (`spend`), and hit is the set of coarse plaquettes near them Source .

How it is paid Inferred : what fills the right-hand side directly is the crossing row $\mathbf{workL} + \mathbf{workD}$ on hit. The other three terms on the left are the accounting of its error.

- $(C/L^2)\, t \sum \mathrm{toll}$ is paid by the $(c_{\mathrm{lin}}/2)\, t \sum \mathrm{toll}$ of the lower bound for fine plaquettes and the penalty. This is why L is taken large.
- The linear term in the deviations is paid by the quadratic of the lower bound for the penalty row.
- $Zt^2 R^4 \# A_{\mathrm{sp}}$ is paid by the linear action of the especially large plaquettes ($t \cdot \beta^{-0.49} \gg t^2 (\log \beta)^8$).

The L^{-2} shrinking. The row sums of the blocking kernel Φ (sums per fine plaquette) are C/L^2, and the column sums (per coarse plaquette) CL^2 (the kernel row sums `Allowance`, :61) Source . Since the coarse curvature is roughly $\sum_p \Phi(p,q) X_p^f$, for linear quantities $\sum_q |\mathrm{curv}_q| \lesssim (C/L^2) \sum_p |\mathrm{curv}_p|$ Inferred . Quadratic actions keep their form under blocking, but the total of a linear action shrinks by L^{-2}. Collecting L^4 fine curvatures with weight L^{-2} in four dimensions is consistent with $\sigma_c \approx L^2 \sigma_f$ for smooth fields Inferred .

05

Assembly, and the factor that comes out of the integral

The remaining components are as follows Source (names and forms of the statements).

Group name	Declaration	Content
E01	`Payments` (:27)	Large coarse plaquettes lie only near especially large seeds, and their number is $\le K \# A_{\mathrm{sp}}$. The number of coarse flags is $\le 4\times$ (the number of plaquettes above the threshold τ)
E03	`Allowance` (:61)	Row sums of the kernel Φ are C/L^2, column sums CL^2
E04	`Maximum` (:83)	$\|\mathbf{vision}\| \le Ct$, $\|M_v\| \le (C/L^2)t$
E08	`Stitches` (:134)	Outside, where no seed is hit, both the coarse and fine fields are small in a local gauge
Smallness outside	`Belly` (:142)	The negative part of the outside rows is $\ge -\varepsilon t^2 \# A$ (for every $\varepsilon > 0$)
E09	Same file	Assembles `Grants` from the above

Extracting a factor from the integral: `Sector.peel` (`YM/Next/FLUCTUATIONS/Sector.lean:229`) Source says that when (1) holds on a region, for the integral over the region

$$\sum_{k} \int_{\text{region}} g \cdot \text{incoming} \;\le\; \text{pref} \cdot e^{-\beta c E t' \# A} \sum_{k} \int_{\text{region}} g \cdot \text{outgoing} \tag{6}$$

holds. outgoing is the exponential of the rows outside the neighbourhood. If β is taken of the order of the fine-step coupling, then $\beta t^2 \approx P^2$ ($P = \exp((\log\log\beta)^2)$), so **a factor of order e^{-cEP^2} per seed** comes out Inferred . This lemma does not say how large the outgoing integral is Source (11-05, 11-11).

P^2 grows faster than any power of $\log\beta$, so the halo volume $L^4 R^4$ and polylogarithmic losses can be paid with this factor (`Delay.inactive_price`, `YM/SectorNext/LARGE_FIELD_REMAINDERS/Delay.lean:120`) Source (an asymptotic statement).

06

What this book's study had seen

What was open in this book's study was the risk that, near configurations whose shape the block map does not change, the gains in the action balance out so that rarity does not come out at each step (the "large-field estimate" row in 28-06). The published construction **answers this in a different form**: it truncates the main part of the representation (the action handed to the coarse step) to linear, puts the difference into activities, and sets up an **inequality uniform over configurations satisfying the gauge section and the region conditions**, (1). The one-step map stays an identity, and the measure does not change. Statements (1)–(5) contain no hypothesis that excludes types of exceptional configuration Source .

What this book had in Lean was the rarity of large plaquettes at a single scale (the Wilson measure of the fine lattice itself) (22-01, from the Chessboard Estimate (Fröhlich–Israel–Lieb–Simon), uniform in the volume). It did not have a configuration-by-configuration inequality at the pushed-forward coarse scale.

07

What has not been read

- The proofs of the three inequalities (`YM/Work/E05` – `YM/Work/E07`). Only the statements and docstrings were read.
- The details of the roles of the shell comparison (`Cops`, group name E02) and of the smallness outside (`Stitches`, E08), and the meaning of the tube fields of `Funds` (`Deep`, `Roll`).

- Whether `Grants` and `peel` lie on the chain of the existence theorem (the witness in the record is `SharpGram.habitat`).
- How the difference between the orientation-averaged action and the crown of a single orientation is counted across steps.

What this chapter can say

Can say	Cannot say
Source In the quantified form (1), $c_E t^2 \# A \leq$ assessed on a region (with the gauge section, the side lower bound and `Funds`). Holds for every order (`grants`)	A claim across steps (it is a one-step, configuration-by-configuration inequality)
Source Lower bound for coarse-plaquette rows (2): positive remainder $c_{\mathrm{dir}} \tau \sigma / 4$ on medium plaquettes	An explicit value of c_E (only existence, depending on L, W)
Source Lower bound for fine plaquettes and the penalty (4), payment for large coarse plaquettes (5)	That this book has read the proofs of (2), (4), (5)
Source The integral over a region yields $e^{-\beta c_E t^2 \# A}$ (`peel`)	The size of the remaining integral (`peel` does not say)
Inferred The positive remainder comes from truncating the coarse action to linear	That the problem of persistent configurations disappears by this truncation alone (what can be said goes only as far as an inequality uniform over configurations satisfying the section and region conditions)

Sources and reproduction

Item	Kind	Source / tool
Order, section, quantifiers, energy per seed	Source	`YM/Pressure/Seeds.lean:48` (`SliceSet`), :89 (`Teller`), :101-127 (`Funds`, `live`, `Serves`, `cash`), :129 (`Grants`)
The three inequalities and components	Source	`YM/Bid/Energy.lean:27` (E01), :61 (E03), :83 (E04), :94 (`Crowds`), :101 (`def E05`), :103 (`Seals`), :110 (`def E06`), :121 (`Lapse`), :130 (`def E07`), :134 (`Stitches`), :142 (`Belly`), :155 (`issues`)
Holding without hypotheses	Source	`YM/Next/FLUCTUATIONS/Grants.lean:24` (`grants`), `YM/Integrated.lean:222`, `YM/Next/FLUCTUATIONS/Bare.lean:64`
Extracting a factor from the integral	Source	`YM/Next/FLUCTUATIONS/Sector.lean:229` (`peel`)

Item	Kind	Source / tool
Asymptotic lemmas	Source	`YM/SectorNext/LARGE_FIELD_REMAINDERS/Delay.lean:62` (`cover_pay`), :120 (`inactive_price`)
Dependency record	Source	`verification/declaration-usage.tsv` (`Teller.Grants`, `Crowds`, `Seals`, `Lapse`, `Sector.peel` are used, witness `SharpGram.habitat`)

Words used in this chapter — seed, number of seeds, region, gauge section, positive remainder, blocking kernel, order bundle. Their meanings are in the A-02 glossary.

Next chapter to read: 11-05 Allocating prices, and polymer sums — the factor that came out is split into marks and combinatorial payments.

Allocating prices, and polymer sums

> This chapter is established knowledge (a reading of the published construction). § 05 is this book's own investigation and § 06 is a map of open questions.

What this chapter shows — How the per-seed factor $e^{-\beta c_E t^2}$ that came out of the region integral is distributed. Half of the guaranteed lower bound $c_E P^2/2$, namely $c_E P^2/4$, is left in the exit activities as a "mark", and the rest pays for the tree expansion of polymers, the covering combinatorics and the polylogarithmic costs. As a result, the pointwise sum of connected polymers containing a seed is bounded by a quantity proportional to $e^{-c_E P^2/4}$.

Chapters assumed — 11-04 Energy per seed (the energy-per-seed constant c_E and `peel`), 7-04 The cluster expansion (the idea of the polymer expansion and tree estimates).

Said first — The formulas of the accounting fields were read in the source, but the proof of the polymer-sum estimate (Ursell coefficients and the tree inequality) has been read only as far as names and types. The declarations of this chapter are reached in the verification record from one of the headline theorems, but we have not checked whether they lie on the chain of the existence theorem.

The order of this chapter

1. Statement — the price fields and the mark
2. Statement — the pointwise sum of polymers containing a seed
3. Outline of the mechanism — why the super-polylogarithmic P is enough
4. What follows — conversion into a rate
5. What this book's study had seen
6. What has not been read

Labels: Source = read in the source of the published construction / Inferred = this book's inference / Assessment = according to the committee's assessment. The other labels (Proved here, Cited, Reading, Lean and so on) mean the same as in the rest of the book (for details see "Labels" in 6-00).

01

Statement — the price fields and the mark

Notation: the reference coupling of the coarse step x, that of the fine step $\beta_f = x + s \log L$ (s is the one-loop slope) (`betaF`), the fine threshold $t = \mathrm{fiery}(x, L)$, and $P = \exp((\log\log\beta_f)^2)$ (`fineMonitor`). Then $\beta_f t^2 = P^2$ Source (by definition). $c_E > 0$ is the energy-per-seed constant (the c_E of 11-04; in the source's wording at this location, μ).

Income. The income of one seed is $\mathrm{gain} = \beta\, c_E\, t^2$ (`credit`, `YM/Push/ABSOLUTE_SUMS/Grants.lean:48`), and it depends on the actual β Source. Under the band condition $\beta \geq u\,\beta_f$, the income is bounded below by

$$\mathrm{gain} = \beta c_E t^2 \;\geq\; \mathrm{hatWin}(c_E, u) = c_E\, u\, P^2 \quad (1)$$

(`Delay.comparison`, `YM/Push/ABSOLUTE_SUMS/Delay.lean:43`; `hatWin` is :28) Source. With $u = 1/2$, the **guaranteed lower bound** is $c_E P^2/2$.

The **price fields** `Buy.quote` (`YM/Push/ABSOLUTE_SUMS/Buy.lean:31`) fill the fields of the accounting type `Costs` (`Mixed.lean:26`) as follows Source. Where it is used, `Target.costs` (`YM/AnalyticNext/CONNECTED_ACTIVITY_JETS_E6/OptionCost.lean:83`), $u = 1/2$.

Field	Value ($u = 1/2$)	Role
gain	$\beta c_E t^2$ ($\geq c_E P^2/2$)	Income of one seed
markSeed (mark)	$\mathrm{hatWin}(c_E/2, 1/2) = c_E P^2/4$	Rarity left in the exit activities
packetRatio	$e^{-c_E P^2/8}$	Polymer ratio (convergence of the tree expansion)
rootHit	$e^{-c_E P^2/16} + \gamma$	Weight of the root of the tree
spendSeed	$C \cdot \mathrm{bill}^5$, $\mathrm{bill} = 1 + R + (\log\beta_f)^2$	Logarithmic cost per seed (polylogarithmic)
cap, extraSeed	$1, 0$	Constants imposed on polymers
optionalHit γ	β_f ($-1/2 < r < 0$)	Additional weight

The source's docstring says: "use half of the guaranteed income as the mark that is extracted" Source (docstring). Here **the guaranteed income is the lower bound $c_E P^2/2$ of (1)**, and the mark $c_E P^2/4$ is half of it. It is not half of the actual income $\beta c_E t^2$ (since β moves within the band).

Sufficient condition `Buy.quoted` (`Buy.lean:67`) Source : fixing $c_E > 0$, $z < 2$, a constant $D \geq 0$ and a degree d, for sufficiently large x, if the logarithmic cost per seed and the logarithm of the polymer constant are at most $\text{fee} = D \cdot \text{bill}^d \cdot P^z$ and γ is small, then the combinatorial condition `Buy` holds. `Target.pay_for` (`OptionCost.lean`) applies it with $d = 5$ Source .

02

Statement — the pointwise sum of polymers containing a seed

The result of the one-step integral is rearranged into a sum over "polymers", finite sets of seeds and letters (such as the addresses of incoming activities) (11-06). The sufficient condition for a geometric series of polymer weights and covering weights is `Packets.income` and `fit` (`YM/Push/ABSOLUTE_SUMS/Charge.lean:22-38`) Source :

$$\text{income}(C, \text{spend}, \text{gain}, m) = C\, e^{(\text{spend}-\text{gain})\, m}, \qquad (\text{covering weight}) = e^{a\, \text{cov} B}. \tag{2}$$

The upper bound on the pointwise sum of connected polymers containing a seed, `Cash.stocks` (`YM/AnalyticNext/CONNECTED_ACTIVITY_JETS_E6/Bulkhead.lean`; `armK` is :41) Source : at each point x, the sum of covering weight $\times$ polymer weight over polymers r that contain x, carry a mark, and have at least m elements is

$$\sum_{r \ni x,\ \#r \geq m} (\cdots) \ \leq\ \text{armK} = 2^{-m}\, e^{-\text{mark}} \cdot \text{mergeScale} \cdot 2 \tag{3}$$

at most this. $\text{mark} = c_E P^2/4$, and mergeScale is a quantity that goes to 0 at the rate of a negative power of β Source (the type of the statement `tempo`).

Which target it enters. `Buy.quote`, `Cash.armK` and `dictionary_fit` are "used" in the verification record, and the witness in the record is `SharpGram.habitat` Source .

03

Outline of the mechanism — why the super-polylogarithmic P is enough

1. **How the income is split.** The field values are mark $c_E P^2/4$, polymer ratio $e^{-c_E P^2/8}$ and tree root $e^{-c_E P^2/16}$ Source (field values). Reading this as "splitting the guaranteed lower bound $c_E P^2/2$" is this book's reading ($1/4 + 1/8 + 1/16 = 7/16 \le 1/2$); it is not written in the source as a single account Inferred. The mark remains just once in the polymer sum (3) as $e^{-\mathrm{mark}}$, and the polymer ratio and root weight pay for the convergence of the tree expansion of connected polymers Inferred.
2. **Polylogarithmic costs.** The halo volume, the number of coverings, Haar losses and so on are of the order of powers of $\log\beta$ per seed, and are collected in the field spendSeed $= C\,\mathrm{bill}^5 \approx C(\log\beta)^{10}$ Source (read the field) / Inferred (reading of what goes into it).
3. **How P grows.** $P = \exp((\log\log\beta)^2)$ grows faster than any power of $\log\beta$ and slower than any positive power of β. So P^2 always beats the cost $D\,\mathrm{bill}^d P^z$ ($z < 2$), while the threshold $t = \beta^{-1/2}P$ goes to 0 (`Delay.cover_pay`, `inactive_price`) Source (asymptotic statements).
4. **Why it is written as proportions.** The fields are all functions of c_E and P, written as lower bounds that do not depend on where β sits in the band. The induction hypothesis has no field that carries a decrease over to the next step Source (`Conv`), and c_E and the rate are chosen afresh at each step. Conditions uniform in the step are paid for on the calendar's side (11-08, 11-07) Inferred.

04

What follows — conversion into a rate

The polymer sum (3) is converted into a "rate" for the exit forced activities Source.

- `eventual_resize` (`YM/PhysicalNext/RG_NEXT_COMPLETE_REPEATABLE_NONLINEAR_STEP/Rent.lean:34`): if the rate $h \le c_E/16$, then for sufficiently large x, $\mathrm{armK} \le c \cdot Ce^{-h\,(t_y/\mathrm{gap}\,\beta)^2}$. The source's docstring says: "updating the strong ledger needs a rate proportional to P^2, not P^δ ($\delta < 2$); read it back from the integrated estimate before dropping to a smaller exponent".
- `Pocket`, `pockets` (same file, :70, :76): for a given upper bound H on the rate, choose rate $\le \min(H, c_E/64)$ and $h = c_E/32$.
- `dictionary_fit` (`Bulkhead.lean:78`): for any $0 \le \delta < 2$ and $A \ge 0$, for sufficiently large x, $\mathrm{armK} \le e^{-AP(x)^\delta}$. The $P(x)$ here is the value at the coarse-side reference coupling x.

The whole chain of conversion is read in 11-07.

05

What this book's study had seen

This book's study thought of the rarity of large fields in the form $e^{-c/g^2} = e^{-c\beta}$, and assumed that the ledger of activities would carry it (the "large-field estimate" in 28-04). The per-seed price of the published construction is e^{-cEP^2}, a much weaker rarity than $e^{-c\beta}$. It suffices because the ledger $e^{-P^{1/5}}$ required by the induction hypothesis is weaker still Inferred . The form of accounting that splits the income in proportions and leaves a mark did not exist on this book's side either.

06

What has not been read

- The proof of the polymer sum (3): Ursell coefficients and the Penrose-type tree inequality (`YM/Push/ABSOLUTE_SUMS/Ursell.lean` , `Graph.lean`), and the dictionary of connected activities (`YM/AnalyticNext/CONNECTED_ACTIVITY_JETS_E6`).
- The concrete lemma that puts the Haar loss into the spendSeed field (11-11).
- All the fields of the combinatorial condition `Buy` .
- The definition of $\mathrm{mergeScale}$, and the details of the statement (`tempo`) that it goes to 0 as a negative power of β.

What this chapter can say

Can say	Cannot say
Source If $\beta \geq \beta_{\mathfrak{f}}/2$, the income $\beta c E t^2 \geq cEP^2/2$ (the guaranteed lower bound)	The value of the actual income (β moves within the band)
Source The mark is $cEP^2/4$, half of the guaranteed lower bound $cEP^2/2$; polymer ratio $e^{-cEP^2/8}$; root $e^{-cEP^2/16} + \gamma$	That the mark is "half of the income" (it is half of the lower bound)
Source Pointwise sum of connected polymers containing a seed $\leq 2^{1-m} e^{-cEP^2/4}$ $\mathrm{mergeScale}$	The content of its proof (this book has not read it)
Source Costs up to D $\mathrm{bill}^d P^z$ ($z < 2$) can be paid for sufficiently large x (`quoted`)	A statement that the Haar loss goes into this field (unconfirmed)

Sources and reproduction

Item	Kind	Source / tool
Income and the guaranteed lower bound	Source	`YM/Push/ABSOLUTE_SUMS/Grants.lean:48` (`credit`), `Delay.lean:28` (`hatWin`), :43 (`comparison`)
Price fields	Source	`YM/Push/ABSOLUTE_SUMS/Mixed.lean:26` (`Costs`), `Buy.lean:31` (`quote`), :67 (`quoted`), `YM/AnalyticNext/CONNECTED_ACTIVITY_JETS_E6/OptionCost.lean:83` (`Target.costs`)
Polymer weights	Source	`YM/Push/ABSOLUTE_SUMS/Charge.lean:22-38` (`income`, `fit`)
Polymer sums	Source	`YM/AnalyticNext/CONNECTED_ACTIVITY_JETS_E6/Bulkhead.lean:41` (`armK`), `stocks`, :78 (`dictionary_fit`)
Conversion into a rate	Source	`YM/PhysicalNext/RG_NEXT_COMPLETE_REPEATABLE_NONLINEAR_STEP/Rent.lean:34` (`eventual_resize`), :70 (`Pocket`), :76 (`pockets`)

Words used in this chapter — price, income, guaranteed lower bound, mark, polymer, polymer ratio, covering weight, polylogarithmic cost. Their meanings are in the A-02 glossary.

Next chapter to read: 11-06 Reorganising activities at the coarse step — polymers are split into those that contain a coarse flag and those that do not, and entered into the exit representation.

Reorganising activities at the coarse step

This chapter is established knowledge (a reading of the published construction). § 05 is this book's own investigation and § 06 is a map of open questions.

What this chapter shows — Where, after the result of the one-step integral has been rearranged as a sum over polymers, each piece goes in the representation at the coarse step. Polymers containing a coarse flag go into the gas of exit forced activities; polymers not containing one go to the side of regular activities. The constants of the latter flow to the vacuum term, and their quadratic parts at the anchor flow to the correction of the coupling.

Chapters assumed — 11-01 (representation and gas), 11-05 (polymers and marks).

Said first — This chapter reads the **classification and the formulas for where things go** in the reorganisation. The map itself by which the values of polymers not containing a coarse flag enter the addresses of regular activities has not been read by this book. That part is written under Inferred and listed in § 06.

The order of this chapter

1. Statement — two kinds of polymer, and the value of the gas
2. Statement — fusing the hard cluster and the soft cluster
3. Outline of the mechanism — where defects that have returned to small field go
4. What could be read, and the map that has not been read
5. What this book's study had seen
6. What has not been read

Labels: Source = read in the source of the published construction / Inferred = this book's inference / Assessment = according to the committee's assessment. The other labels (Proved here, Cited, Reading, Lean and so on) mean the same as in the rest of the book (for details see "Labels" in 6-00).

01

Statement — two kinds of polymer, and the value of the gas

The quantities produced by the one-step integral are rearranged into a sum over polymers (`Molecule`: a finite set of seeds and letters). Polymers are split in two by the set of coarse flags of the combined seed types of a polymer M (`roster`) (`YM/AnalyticNext/CONNECTED_ACTIVITY_JETS_E6/Colony.lean:26-27`) Source .

$$\mathrm{ThickP} = \{M : \mathrm{roster}(M).\mathrm{flags} \neq \emptyset\}, \qquad \mathrm{ThinP} = \{M : \mathrm{roster}(M).\mathrm{flags} = \emptyset\} \tag{1}$$

(both with non-empty support). We call polymers containing a coarse flag "flagged polymers" and those not containing one "unflagged polymers".

The gas of flagged polymers (`banked`, :43) Source : for a finite set $\mathcal{A}$ of flagged polymers, it takes the value of the gas of those with pairwise disjoint supports only when the union of their flag inventories equals the flags of the coarse field $\mathbf{flags}_{t_m}(V)$ exactly; otherwise it is 0. This has the same form as the gas of the coarse-step representation (equation (6) of 11-01 § 03).

The values of **unflagged polymers** (`entries`) are arranged as a separate gas (`ignores`, `inlet`), avoiding the supports of the flagged polymers Source (form of the definition).

The vacuum term (`Collection.vac`, `YM/AnalyticBridge/VERIFIED_RESET_STEP/Evaluation.lean:29`) Source : the exit vacuum constant is

$$v_{\mathrm{out}} = \tau - \frac{1}{|\Lambda_m|} \sum_i \log\text{-value}_i(1) \tag{2}$$

Here τ is a constant coming from the normalisation of the integral, and $\log\text{-value}_i(1)$ is the value at the flat configuration of each piece on the regular-activity side.

02

Statement — fusing the hard cluster and the soft cluster

The addresses of the exit forced activities are made by joining two clusters Source .

- **Hard cluster**: the values of flagged polymers. The pointwise sum is bounded by $\mathrm{strong} = C\, e^{-r(t y / \mathrm{gap}\ \beta)^2}$ (r is the rate, y the reference coupling of the fine step;

`Pairing.strong`, `YM/Progress/DIRECTED_RECIPE/Economy.lean`).

- **Soft cluster**: small-field logarithmic pieces with support on the unhealthy part of the coarse step. The sum is $\mathrm{mild} = C(1+B)^d P^e \,\mathrm{gap}^p$ $(p > 0)$, which becomes small at the rate of a positive power of β (`Pairing.mild`, same file).

The fusion condition `Rates` (`YM/Progress/DIRECTED_RECIPE/Tender.lean:33`) and the theorem `tender` (:124) Source : for an exponent $k \geq 0$ (κ in the source), a hard sum a and a soft sum $b \leq 1$, if

$$e^{-k}\left(e^{\,e^{k}a+2b}-1\right) \;\leq\; w \quad (3)$$

(the left-hand side is `Bid.quote`, `YM/Bid/Kitchen.lean:18`) and the covering-number condition $(2B+1)^4(e^k a + 2b) \leq 1$ hold, then the fused activities satisfy `KBound` with ledger w. `Pairing.currency` (`Economy.lean:76`) satisfies `Rates` with $w = e^{-k}$, $k = \mathrm{kGoal} = \mathrm{rate} \cdot (t_y/\mathrm{gap}\,\beta)^2$, $a = \mathrm{strong}$, $b = \mathrm{mild}$. Since the rate satisfies $r > \mathrm{rate}$, $e^k\,\mathrm{strong} = Ce^{-(r-\mathrm{rate})(\cdots)^2} \to 0$ Source (statement read) / Inferred (reading of why $e^k a$ becomes small).

Each address contains a coarse flag (the `permit` of forced activities), so we read it as containing at least one hard piece, and e^{-k} is applied once Inferred .

03

Outline of the mechanism — where defects that have returned to small field go

Consider the case where a plaquette (seed) that was large at the fine step does not exceed the threshold t_m at the coarse step, that is, **returns to small field at the coarse step**. A polymer containing that seed contains no coarse flag, so it is an unflagged polymer and does not enter the gas of forced activities. The gas is 1 when the flags are empty (`gas_one`), so there is no place to keep it as a forced activity in the first place Source .

We read where it goes as follows Inferred .

1. The value of an unflagged polymer goes to the side of regular activities (which live only where healthy).
2. Its value at the flat configuration moves into the vacuum term v by equation (2).

3. Its quadratic part in the curvature direction at the anchor moves into β as the correction of the coupling Δ (`Averaging.delta`, 11-09 § 06).
4. The rest is normalised to "0 at the flat configuration, 0 in the quadratic of the affine directions" (`rebate`) and enters the regular-activity ledger $\hat{g}^{11/5}$. Because of the seed's mark $e^{-cEP^2/4}$, it falls well below the ledger.

If it is still large at the coarse step, it becomes the inventory of a new forced activity as a flagged polymer, and need only fit within the ledger $e^{-P^{1/5}}$ (11-07). This split handles the case distinction of whether a defect has returned with a single local test: whether a coarse flag is contained Inferred .

04

What could be read, and the map that has not been read

Component	Level	Location
Definition of the two kinds of polymer (1)	Source	`Colony.lean:26-27`
Value of the gas of flagged polymers (exact agreement with the flags)	Source	`Colony.lean:43` (`banked`)
Values of unflagged polymers enter the Ursell terms and, if non-zero, are healthy at the coarse threshold	Source	`Fire.lean:43` (`trained`), `Street.lean:68` (`clean_claim`) (same location)
Formula (2) for the vacuum term	Source	`Evaluation.lean:29`
Fusion condition (3) and `currency`	Source	`Tender.lean:33`, :124, `Economy.lean:76`
The map by which values of unflagged polymers enter the addresses of regular activities (log-values)	Inferred	From `entries` onward. Not read
Handling of the addresses of incoming activities (footprint `forcedFoot`, covering cost `forcedExtra`)	Source definitions only	`Inherited.lean:150`, :162

Which target it enters. `Collection.vac` is "used" in the verification record, with witness `SharpGram.habitat`. $v_F = \mathrm{vac}$ enters the conclusion of the small-field one-step theorem `Return.reset`, and `reset` lies on the chain of the existence theorem (11-09) Source .

05

What this book's study had seen

Because this book's study fixed large-field regions thereafter, it needed separate treatment for the case where a defect returns to small field at the coarse step. The

published construction makes this case distinction by a local test of whether a coarse flag is contained, and puts what has returned onto the vacuum term and the flow of the coupling Inferred . This pairs with the mechanism by which the correction of the coupling is fixed as a coefficient in the representation (10-03).

06

What has not been read

- The map that puts the values of unflagged polymers into the addresses of regular activities (from `entries` of `Colony` onward, `YM/AnalyticNext/CONNECTED_ACTIVITY_JETS_E6`).
- The handling of incoming activities (`Sales`, `ratesCombine`, `OptionCost.lean`) and what follows.
- The proofs of the reorganisation lemmas (`Marlin`, `Trio`, `Meadow` in `YM/PhysicalNext/RG_NEXT_COMPLETE_REPEATABLE_NONLINEAR_STEP`).
- The whole construction that rebuilds new forced activities after one step and fits them into the ledger $e^{-P_m^{1/5}}$ (11-11).

What this chapter can say

Can say	Cannot say
Source Polymers split in two by whether they contain a coarse flag, and the gas of flagged polymers has a value only when it agrees exactly with the flags	The map by which values of unflagged polymers enter the addresses of regular activities (not read)
Source The exit vacuum term is $\tau - \sum \log\text{-value}(1)/\lvert\Lambda\rvert$	An estimate of the content of τ (normalising determinants and so on)
Source `currency` satisfies the fusion condition (3) for the hard and soft clusters	The details of the proof of fusion
Inferred Defects that return to small field at the coarse step flow into the vacuum term and the correction of the coupling	That this flow is written as a statement

Sources and reproduction

Item	Kind	Source / tool
Two kinds of polymer, value of the gas	Source	`YM/AnalyticNext/CONNECTED_ACTIVITY_JETS_E6/Colony.lean:26`, :27, :43

Item	Kind	Source / tool
Addresses of incoming activities	Source	`YM/AnalyticNext/CONNECTED_ACTIVITY_JETS_E6/Inherited.lean:150` (`forcedFoot`), :162 (`forcedExtra`)
Vacuum term	Source	`YM/AnalyticBridge/VERIFIED_RESET_STEP/Evaluation.lean:29` (`vac`)
Fusion	Source	`YM/Progress/DIRECTED_RECIPE/Economy.lean:47` (`kGoal`), :76 (`currency`), `Tender.lean:33` (`Rates`), :124 (`tender`), `YM/Bid/Kitchen.lean:18` (`quote`)

Words used in this chapter — polymer, flagged polymer, unflagged polymer, hard cluster, soft cluster, vacuum term, correction of the coupling. Their meanings are in the A-02 glossary.

Next chapter to read: 11-07 Converting between ledgers — the strong estimate at the exit is moved to the weak ledger of the induction hypothesis.

Converting between ledgers, and persistence across steps

This chapter is established knowledge (a reading of the published construction). § 05 is this book's own investigation and § 06 is a map of open questions.

What this chapter shows — The final part of the chain of conversions from price to rate. The forced activities at the exit of one step are first bounded in the strong form $e^{-\text{rate}\cdot P_y^2}$, and this is weakened in two ways: to $e^{-dP_y^2}$, recorded in the one-step certificate, and to $e^{-P_x^{1/5}}$, carried by the induction hypothesis. We then read how defects that persist over many steps are handled under these ledgers.

Chapters assumed — 11-00 § 03 (the two ledgers), 11-05 (marks and conversion into a rate), 11-06 (how the exit activities are made).

Said first — The weak ledger is also a function of step m. The conditions for making things hold uniformly in the step are paid for on the side of the calendar (the entry for β, the lower bound on the sides, the choice of rate). The estimate of incoming activities does not enter the formula for the exit rate, but it is used on the side of the one-step hypotheses to make sums converge. The conversion lemmas of this chapter are reached in the verification record from one of the headline theorems, but we have not checked whether they lie on the chain of the existence theorem.

The order of this chapter

1. Statement — the chain of conversions from price to ledger
2. Statement — the one-line inequality that lowers the exponent
3. Carrying the one-step certificate
4. Outline of the mechanism — defects that persist over many steps
5. What this book's study had seen
6. What has not been read

Labels: Source = read in the source of the published construction / Inferred = this book's inference / Assessment = according to the committee's assessment. The other labels (Proved here, Cited, Reading, Lean and so on) mean the same as in the rest of the book (for details see "Labels" in 6-00).

01

Statement — the chain of conversions from price to ledger

Notation: the reference coupling of the fine step y, that of the coarse step x, $P_y = P(y) = \exp((\log\log y)^2)$, and the threshold t_y. Let c_E be the energy-per-seed constant. The conversion within one step is the following chain Source (statements at each stage).

Order	Quantity	Location
1	$c_E t^2 \#A \le$ assessed on a region	`Teller.Grants` (11-04)
2	Half of the guaranteed lower bound $c_E P_y^2/2$ becomes the mark $c_E P_y^2/4$	`Buy.quote` (11-05)
3	Pointwise sum of polymers containing a seed $\le \mathrm{armK} \propto e^{-c_E P_y^2/4}$	`Cash.armK`
4	The estimate with rate $h \le c_E/16$, $\mathrm{armK} \le c\,C e^{-h(t_y/\mathrm{gap}\,\beta)^2}$	`eventual_resize`
5	Choose $h = c_E/32$ and rate $\le c_E/64$	`Pocket`, `pockets`
6	Fused exit activities $\le e^{-\mathrm{kGoal}}$, $\mathrm{kGoal} = \mathrm{rate}\cdot(t_y/\mathrm{gap}\,\beta)^2$	`tender`, `Pairing.currency` (11-06)
7	$e^{-\mathrm{kGoal}} \le e^{-dP_y^2}$ and $\le e^{-P_x^{1/5}}$	`Time.late_force` (§ 02 below)

For β in the band, $t_y/\mathrm{gap}\,\beta$ is of the order of a constant multiple of P_y, so kGoal is of the order of $\mathrm{rate}\cdot P_y^2$ Inferred (from the comparison $P_y \le 2\,t_y/\mathrm{gap}\,\beta$ inside the proof of `late_force`).

Summary of the one-step exit: `Time.now` (`YM/AnalyticBridge/VERIFIED_RESET_STEP/Opening.lean:84`) Source bundles the exit ledger of regular activities $\mathrm{forecastE} = \mathrm{credit}\cdot\hat{g}_m^{11/5}$ and the exit estimate of forced activities (`forecastK`, the strong of the hard cluster), and from the same estimate produces two things (relaxing the ledger with `K_rises`): the induction-hypothesis form $\|K\| \le e^{-P_m^{1/5}}$, and the one-step-certificate form $\|K\| \le e^{-d\,P_{m+1}^2}$.

02

Statement — the one-line inequality that lowers the exponent

`Time.late_force` (`Opening.lean:19`) Source : for $d > 0$ and a constant c, for sufficiently large x, for every rate satisfying $d \le \mathrm{rate}/4$ and every β in the band of $y = x + c$, $r = e^{-\mathrm{kGoal}(y,\beta)}$ satisfies

$$r \le e^{-d\,P_y^2}, \qquad r \le e^{-P_x^{1/5}} \tag{1}$$

The d here is the same as the one-step certificate's coefficient in § 01 and § 03 (`Time.now` checks $d \le \mathrm{rate}/4$ for the coefficient chosen by the calendar and uses `late_force`) Source . The second half comes from `Postage.saved` (`YM/Next/SCHEDULE/Salary.lean:178`) Source : when two scales are close (`Alike`: including $P_x \le 2P_y$ and so on), for sufficiently large y

$$e^{-d\,P_y^2} \le e^{-P_x^{1/5}}. \tag{2}$$

The proof is the single line $P_x^{1/5} \le (2P_y)^{1/5} \le 2P_y \le dP_y^2$, using the exponent $1/5 \le 1$ Source . A form in which any exponent below 2 will do is stated elsewhere: `Cash.dictionary_fit` (`YM/AnalyticNext/CONNECTED_ACTIVITY_JETS_E6/Bulkhead.lean:78`) gives $\mathrm{armK} \le e^{-AP(x)^{\delta}}$ for any $0 \le \delta < 2$ ($P(x)$ is the coarse-side value) Source .

Note: `late_force` does not say that r is an upper bound for the actual activities. That is produced separately by items 1–6 of the chain in § 01 Source .

03

Carrying the one-step certificate

The field `forced` of the one-step certificate `Directed.Move` (`YM/PCalc/Band.lean:49`) states $\|K\| \le e^{-d\,P_{m+1}^2}$ for the forced activities of the representation u at step m. P is the value at the **fine step** $m+1$ Source .

The induction hypothesis `Conv` and the input conditions of the one-step theorem (`Program.advances`, :65) carry only the weak ledger; the strong ledger does not enter Source . The lemma that records the strong ledger for each Move, `Directed.holding` (:92), is **outside the dependencies** Source (record). On the chain of the existence theorem, `Roof.Lyrics.supplier` (`YM/FullVolume/C/Songbook.lean:76-89`) packs the

Move from the one-step result into the step record (`tunes.next`) and carries it along Source . Who uses `Move.forced` at later steps, this book has not checked.

04

Outline of the mechanism — defects that persist over many steps

What can be read Source .

1. The large-field part of the induction hypothesis at each step m is only "the covering-weighted sum of forced activities $\le e^{-P_m^{1/5}}$", and it **has no field for the history of regions** (`Rep`, `Conv`, `YM/RG/History.lean:165-218`).
2. Forced activities are different from 0 only on the flags of that step (`permit`, `admissible`).
3. Wherever the weight of the partition of unity is not 0, the flags of the fine step always touch a seed of this step (`flagged`).
4. The formulas for the exit rate (kGoal, strong, mark) are functions of c_E, P and β, and **the estimate of incoming activities does not enter these formulas** (`Economy.lean:40-47`, `Threshold.lean:66`).
5. The estimate of incoming activities is used on the **hypothesis side** of one step: in the field `intake` of `Target.Invoices` (`YM/AnalyticNext/CONNECTED_ACTIVITY_JETS_E6/Issue.lean:73-88`) and in the sum condition `Sales`, which includes the covering cost `forcedExtra` of incoming addresses. It is used to make sums converge.
6. Energy per seed (`Grants`) is stated for all configurations satisfying the gauge section and the region (those satisfying the global region condition). Configurations that keep just exceeding the threshold of each step at every step enter the same inequality.

Assembling the reading Inferred : a defect that persists over many steps is recounted as a seed at each step, receives a new price e^{-cEP^2} from the action difference of that step, and if still large at the coarse step becomes the inventory of a new forced activity as a flagged polymer. The rarity paid at the previous step is relaxed to the weak ledger (`K_rises`), and at the next step it is extracted afresh from the action difference of that step. Along the chain of conversions the constants decrease by some fraction at each step, but the induction hypothesis has no field that carries the decrease over to the next step Source (`Conv`), and cE and the rate are chosen afresh at each step.

Where conditions uniform in the step are paid for Source : the weak ledger $e^{-P_m^{1/5}}$ and the threshold t_m are both functions of step m. The "sufficiently large x" conditions needed for this conversion to hold at every step are paid for step by step as constants

of the calendar: the entry for β (`Delays.betaEntry`), the lower bound on the sides (`floorL`), the choice of rate (`Pocket`), and the exit conditions (`Opened`). For details see 11-08.

In summary, for persistent defects the published construction does not build a single estimate uniform in the number of steps; it builds, at every step, an estimate that returns to a weak ledger of the same form, and pushes the part uniform in the step onto the calendar's conditions Inferred .

05

What this book's study had seen

This book's study stacked up and fixed large-field regions step by step, and thought it necessary to keep the rarity of defects that persist near configurations whose shape the block map does not change uniform in the number of steps (the "large-field estimate" in 28-06). The published construction **answers this in a different form**: it keeps no history of regions, recounts seeds at every step, and, with an inequality uniform over configurations satisfying the gauge section and the region conditions, produces a factor per seed at every step. It does not change the measure.

The components this book's study lacked were the chain of conversions of this chapter and the construction of "two ledgers" Inferred . Replacing this book's ledger e^{-c/g^2} by "a weak ledger and rebuilding at every step" is material for the comparison chapter 14-04.

06

What has not been read

- The whole construction that rebuilds new forced activities after one step and fits them into the ledger (`YM/PhysicalNext/RG_NEXT_COMPLETE_REPEATABLE_NONLINEAR_STEP`, `V9_JOINT_ENDPOINT_LOAD`, `ANNULUS_RATIO_INSERTIONS`). The balancing of the reading assembled in § 04 lies here.
- How incoming activities are handled inside the one-step integral (`Sales`, `ratesCombine`, from `OptionCost.lean:39` onward).
- Where `Move.forced` is used at later steps.
- That the coarse action subtracted at step m has the same path data as the fine action at step $m-1$, so that action differences are not counted twice across steps. The form including the difference between the orientation-averaged action and the action of a single orientation has not been checked.

- Whether `late_force` and `saved` lie on the chain of the existence theorem (the witness in the record is `SharpGram.habitat`).

What this chapter can say

Can say	Cannot say
Source The exit estimate $e^{-\mathrm{kGoal}}$ is at most both $e^{-dP_y^2}$ ($d \le$ rate$/4$) and $e^{-P_x^{1/5}}$ (`late_force`)	That $e^{-\mathrm{kGoal}}$ is itself an upper bound for the activities (another lemma provides that)
Source If $P_x \le 2P_y$, then $e^{-dP_y^2} \le e^{-P_x^{1/5}}$ (the proof uses exponent ≤ 1). The form with any $\delta < 2$ is `dictionary_fit`	That "1/5 has some meaning"
Source The induction hypothesis carries only the weak ledger, and has no field for the history of regions	Where the strong ledger is used at later steps (not checked)
Source The estimate of incoming activities does not enter the formula for the exit rate, and is used in the one-step hypotheses (intake of `Invoices`, `Sales`)	That the estimate of incoming activities is not needed for one step (it is used on the hypothesis side)
Source Conditions uniform in the step are paid for on the side of the calendar's constants (entry for β, lower bound on the sides, choice of rate)	That no condition uniform in the step is needed (it is paid for on the calendar's side)

Sources and reproduction

Item	Kind	Source / tool
Conversion that lowers the exponent	Source	`YM/AnalyticBridge/VERIFIED_RESET_STEP/Opening.lean:19` (`late_force`), :84 (`now`), `YM/Next/SCHEDULE/Salary.lean:178` (`saved`), `YM/Next/SCHEDULE/Dues.lean:57` (`w`), :118 (`Alike`)
Any exponent $\delta < 2$	Source	`YM/AnalyticNext/CONNECTED_ACTIVITY_JETS_E6/Bulkhead.lean:78` (`dictionary_fit`)
Exit rate	Source	`YM/Progress/DIRECTED_RECIPE/Economy.lean:47` (`kGoal`), `YM/PhysicalNext/RG_NEXT_COMPLETE_REPEATABLE_NONLINEAR_STEP/Rent.lean:34`, :70, :76
One-step certificate and carrying it	Source	`YM/PCalc/Band.lean:49` (`Move`), :65, :92 (`holding`, outside the dependencies), `YM/FullVolume/C/Songbook.lean:76` (`supplier`)

Item	Kind	Source / tool
Where the estimate of incoming activities is used	Source	`YM/AnalyticNext/CONNECTED_ACTIVITY_JETS_E6/Issue.lean:73` (`Invoices`)

Words used in this chapter — ledger, strong estimate, weak ledger, rate, one-step certificate, persistent defect, calendar. Their meanings are in the A-02 glossary.

Next chapter to read: 11-08 The calendar — the order in which constants are chosen — how the constants that pay for conditions uniform in the step are chosen.

The calendar — the order in which constants are chosen

> This chapter is established knowledge (a reading of the published construction). § 05 is this book's own investigation and § 06 is a map of open questions.

What this chapter shows — The statements of the one-step estimates stack conditions of the form "if L is sufficiently large", "if W is sufficiently large", "if x is sufficiently large". The calendar is the arrangement of in what order, and to which constants, these "sufficiently" conditions are charged. The order is: step constants → block side → margin of the regions → constant of the energy lower bound → rate → entry for the coupling.

Chapters assumed — 10-05 Why the sides grow, and the calendar, 8-02 The set-up, 11-04 and 11-07 (statements carrying "sufficiently large").

Said first — The calendar's types fix the order of dependence of the constants by type, but, as the source's docstring says, the types themselves do not check "that the renormalisation argument actually fits that order". That is the job of the proofs on the side of the one-step theorem. The reason the sides grow (dividing the one-step error by $\log b$) is the subject of Part 10; this chapter writes only how it looks from the large-field side.

The order of this chapter

1. Statement — the calendar and its fitting conditions
2. Statement — the types for the order of choosing constants
3. Statement — the order of large-field prices (pricing_order)
4. Outline of the mechanism — why it wins even though c_E depends on L
5. What this book's study had seen
6. What has not been read

Labels: Source = read in the source of the published construction / Inferred = this book's inference / Assessment = according to the committee's assessment. The other labels (Proved here, Cited, Reading, Lean and so on) mean the same as in the rest of the book (for details see "Labels" in 6-00).

01

Statement — the calendar and its fitting conditions

Set-up `Schedule` (`YM/Lattice/Scale.lean:28`) Source : block ratios b_m (odd, ≥ 3) and volume periods L_p (multiples of 4, strictly increasing). $D_N = \prod_{k<N} b_k$, spacing $a_m = 1/D_m$. The condition `Gentle` (:129) is $\log b_m / \log D_m \to 0$.

Calendar `Calendar` (`YM/RG/History.lean:64`) Source : the set-up, the choice of kernel, the reference coupling $\bar\beta$, the table of step constants `stage`, the table of sides `factor`, θ, and the number of convolutions J (≥ 4). From these the per-step quantities are fixed ($s = 11C_g/(24\pi^2)$ is the one-loop slope):

$$\hat\beta_m = \bar\beta + s\log\tfrac{1}{a_m}, \quad P_m = e^{(\log\log\hat\beta_m)^2}, \quad t_m = \hat\beta_m^{-1/2}P_m, \quad B_m = \lceil(\log\hat\beta_m)^2\rceil. \tag{1}$$

Main fields of the **fitting condition** `Calendar.Fits` (:97) Source :

- `mesh` : `Gentle`. `sameFactor` : b_m is the $J(m+1)$-th entry of the table of sides. `monotoneL`, `mono` : the sides and J are non-decreasing. `stepSlow` : $J(m+1) \leq J(m) + 1$. `deep` : $J \to \infty$.
- `delayed` : $\hat\beta_m > 1$. `smallField` : $t_m \leq 1$.
- `sizeT` : side of the lattice at step $m \geq 100(B_m + 1)$. `fitsPatch` : side of the fine lattice ≥ 100(patch radius $+1$).
- `driftSmall` : drift$/\log b_m \leq J(m+1)^{-2}$. `logCost` : $\log(1 + \text{sum of step constants})/\log b_m \leq J(m+1)^{-2}$.

The last two are the content of the condition that the sides grow: the logarithmic cost of the step constants and the deviation from one loop are divided by $\log b_m$ and made smaller than J^{-2}. Together with $J \to \infty$, this gives $b_m \to \infty$ Source (read the fields) / Inferred (reading of the derivation).

02

Statement — the types for the order of choosing constants

`YM/Price/Intent.lean` fixes the order in which constants are chosen by nesting types `Source` . The source's headings say "the order and constants chosen before the block ratios", "holds for any constants already chosen", and "the types themselves do not check whether the renormalisation argument fits that order of dependence" `Source` (docstrings).

Order	Type	What it fixes
1	`Sheet` (:26)	The choice of kernel, the table of step constants profile, the first index, the table of lower bounds on the sides floorL (before the block ratios)
2	`Rolls` (:50)	Sides L_j. For each j, `Large` (:39): odd, ≥ 3, $\geq \mathrm{floorL}(j)$, the one-step movement (drift) at most $(s/4)\log L$ (`kick`) and $\mathrm{drift}/\log L \leq j^{-2}$ (`slow`), $\log(1+\text{constants})/\log L \leq j^{-2}$. L_j is non-decreasing
3	`Protection` (:88)	θ
4	`Delays` (:115)	The entry for the coupling betaEntry (after the sides) and so on
5	`Bar` (:123)	The reference value $\bar{\beta}$ (> 1, at least the first entry `ready` (:126), and $x^{-1/2}e^{(\log\log x)^2} \leq 1$ for all x above it `threshold` (:127))
6	`Wait` (:134)	Waits and windows, minimum volume
7	`Levels` and `Paid` (:145)	The calendar. `pays` (:154): $\mathrm{betaEntry}(J(m+1)) \leq \hat{\beta}_m$. `volume`: lower bound on the volume side

`Paid.pays` is where the "entry for β" is paid by the reference coupling of the step. The entry value itself (`Delays.betaEntry`, :116) is extracted as an existence from a bundle of properties that "hold for sufficiently large x" (`open_shop`); there is neither a formula nor a number for it in the source `Source` . From the reference-value condition `Bar.threshold` alone, $\bar{\beta}$ is at least about 9×10^5 `Reading` (the larger root of $\frac{1}{2}\log x = (\log\log x)^2$, $x \approx 8.97 \times 10^5$, found numerically). `logCost` and `driftSmall` of `Calendar.Fits` are the fields of `Large` read with $L = b_m$ `Source` .

03

Statement — the order of large-field prices (pricing_order)

`Bid.pricing_order` (`YM/Bid/Energy.lean:173`) `Source` : if energy per seed `Grants` holds for all (j,i), then there is a table of lower bounds on the sides k such that, for any table of lower bounds n at least k, any choice of sides fitting it (`Rolls`), any θ, and any choice of entries V, there is a W that raises only the entry ($V.\mathrm{betaEntry} \leq W.\mathrm{betaEntry}$, other fields the same), and the price condition (`Ringing`) holds up to any height.

The i in $\exists i\ \forall L \geq i$ of the `Grants` statement becomes the side lower bound k (including the N of the lower bound for fine plaquettes and the penalty). Since `Grants` holds for every order (`Next.Fluctuations.grants`), this ordering comes out without hypotheses as `pressure_pricing` (`YM/Next/FLUCTUATIONS/Grants.lean`) Source .

We read the order in which the one-step large-field constants are chosen as follows Source (read the dependencies of each type and field) / Inferred (arranging them in one line is this book's doing).

$$\underbrace{L}_{\text{side}} \to \underbrace{W}_{\text{region margin}} \to \underbrace{cE}_{\text{energy lower bound}} \to \underbrace{\text{rate},\ h}_{\text{rate (Pocket)}} \to \underbrace{g \leq \text{rate}/4}_{\text{certificate rate}} \to \underbrace{\text{betaEntry}}_{\text{coupling entry}} \Big($$

- L: at least the i of `Grants` (makes the fine-plaquette row beat the cross term $(C/L^2)t$).
- W: the $\forall^\infty W$ of `Grants`. The penalty row pays for sufficiently large deviations.
- cE: the $\exists cE$ of `Grants` (κ in the source). It may depend on L and W.
- Rate: `Pocket` gives $h = cE/32$ and rate $\leq cE/64$ (11-05 § 04).
- g: the coefficient of the one-step certificate's ledger (`PostCost`, `YM/PCalc/Loop.lean:81`). `Time.now` takes $g \leq \text{rate}/4$ as a hypothesis (`Opening.lean:84`).
- betaEntry: all the "if x is sufficiently large" conditions above are paid here together. The exit condition `Opened` (`Opening.lean:45`) says that the one-step price can be paid for all x above the entry, and `open_shop` raises the entry to produce it Source .

04

Outline of the mechanism — why it wins even though cE depends on L

1. The constant of the energy lower bound cE depends on L and W. The losses (halo volume L^4R^4, polylogarithmic factors) also depend on L Source . We read the Haar loss as being of the same type, but have not checked where that lemma is Inferred .
2. But the price per seed is cEP^2, and $P = \exp((\log\log\beta)^2)$ grows with β faster than any power of $\log\beta$. If L is held fixed and the entry for β is raised, cEP^2 beats every loss depending on L Inferred .
3. Raising the entry for β becomes, on the calendar, the condition "the reference coupling of step m, $\hat{\beta}_m$, is at least the entry" (`pays`). Since $\hat{\beta}_m$ increases with $\bar{\beta}$ and

$\log(1/a_m)$, it is paid by taking $\bar{\beta}$ large (`Bar`) and by $\hat{\beta}$ being larger at finer steps Inferred.

4. The sides L_j grow with the step (`Rolls.grow`). The conditions of `Large` are conditions on ratios relative to the constants of step j, so even if the step constants get worse with the step, the sides are grown faster and absorb them Source (read the fields) / Inferred (reading of the meaning).

This whole ordering is the content of "conditions uniform in the step are paid for on the calendar's side" in 11-07. Rebuilding the weak ledger $e^{-P_m^{1/5}}$ at every step goes together with choosing the entry, the sides and the rate afresh at every step.

05

What this book's study had seen

This book's study also had three moves as candidates: growing the sides, fixing the step constants first, and taking the ledger weak (28-05). Under a design with fixed sides, it was looking for a way to keep the rarity of large fields uniform in the step. The published construction uses all three and fixes the order of dependence of the constants by type. This book had nothing corresponding to these "types for the order of choice".

06

What has not been read

- The content of the price condition `Ringing` (`Stamp` at `YM/Pressure/Seeds.lean:155`, `Ringing` at :161).
- The explicit form of the condition that $\bar{\beta}$ be taken large (what can be said from the statements is only $\bar{\beta} > 1$ and that it is at least the entry).
- Which estimates require the window of `Wait` (`window`) and the lower bound on the volume `volume`.
- This book has not traced along the chain that a concrete calendar satisfying the whole calendar exists (the construction of a `Calendar` satisfying `Fits`).

What this chapter can say

Can say	Cannot say
Source The calendar fixes $\hat{\beta}_m$, P_m, t_m, B_m and the ledgers step by step, and `Fits` imposes conditions on the sides, constants and lattice size	That the types themselves check the order of dependence of the renormalisation argument (denied by the source)
Source The order of the constant types is Sheet → Rolls → Protection → Delays → Bar → Wait → Levels, Paid	The roles of all the fields of each type
Source `pricing_order`: builds the side lower bound from `Grants` and returns a choice in which only the entry is raised	The content of `Ringing`
Inferred Large-field constants are chosen in the order L → W → c_E → rate → g → entry, and "sufficiently large x" is paid at the entry	That this order is written in a single declaration (it was arranged from the dependencies of each type)

Sources and reproduction

Item	Kind	Source / tool
Set-up	Source	`YM/Lattice/Scale.lean:28` (`Schedule`), :129 (`Gentle`)
Calendar and fitting conditions	Source	`YM/RG/History.lean:64` (`Calendar`), :79-86 (per-step quantities), :97 (`Fits`)
Types for the order of choosing constants	Source	`YM/Price/Intent.lean:26` (`Sheet`), :39 (`Large`), :50 (`Rolls`), :115 (`Delays`), :123 (`Bar`), :145 (`Paid`), :154 (`pays`)
Order of prices	Source	`YM/Bid/Energy.lean:173` (`pricing_order`), `YM/Next/FLUCTUATIONS/Grants.lean:32` (`pressure_pricing`)
Certificate rate and exit condition	Source	`YM/PCalc/Loop.lean:81` (`PostCost`), `YM/AnalyticBridge/VERIFIED_RESET_STEP/Opening.lean:45` (`Opened`), :84 (`now`)

Words used in this chapter — calendar, set-up, block ratio, lower bound on the sides, entry for the coupling, rate, region margin. Their meanings are in the A-02 glossary.

Next chapter to read: 11-09 The small-field step — the Gaussian integral away from the seeds, and the ledger of regular activities.

The small-field step

This chapter is established knowledge (a reading of the published construction). § 08 is this book's own investigation and § 09 is a map of open questions.

What this chapter shows — The part of the one-step integral that treats the region away from the seeds as a Gaussian integral. The push-forward value is rewritten into the coarse-step representation, without approximation, through three identities, and fitted into the regular-activity ledger $\hat{g}^{11/5}$. The key is to replace the covariance by a windowed approximate inverse and to keep its error inside the integrand.

Chapters assumed — 11-01 (representation and ledgers), 11-03 (regions), 9-03 Two kinds of gauge fixing, 10-02 The one-step map and the one-loop computation.

Said first — What is linked by identities is the one-step **representation**. The ledgers (upper bounds on regular and forced activities) are inequalities; this does not mean that the estimates use no approximation. This book has not read the content of the expansion (Ursell coefficients, cumulants) or the lemmas that reorganise the early ledger into the exit ledger. That the stationary point agrees with the coarse action up to fourth order is carried in the statements as a **condition**, and another theorem supplies it.

The order of this chapter

1. Statement — linking the one-step representation by identities
2. Coordinates — tree gauge and relative coordinates
3. The Gaussian part — precision, gap, windowed approximate inverse, decay
4. The stationary point — agreement with the coarse action is carried as a condition
5. Expansion of corrections, and the early representation
6. Extracting the marginal terms
7. The ledger — why $\hat{g}^{11/5}$
8. What this book's study had seen
9. What has not been read

Labels: Source = read in the source of the published construction / Inferred = this book's inference / Assessment = according to the committee's assessment. The other labels (Proved here, Cited, Reading, Lean and so on) mean the same as in the rest of the book (for details see "Labels" in 6-00).

01

Statement — linking the one-step representation by identities

Notation: the representation at step $m+1$ is Y (coupling $\beta = \beta_{m+1}$); the reference coupling at step m is $x = \hat{\beta}_m$; $\hat{g} = x^{-1/2}$, $P = \exp((\log\log x)^2)$; the threshold $t = \hat{g}P$ (the coarse-side threshold, corresponding to t_m of 9-07 and τ of 11-03; correspondence of threshold letters in 11-01); and the covering scale $B = \lceil (\log x)^2 \rceil$. The one-step kernel is $T_\beta(U,V) \propto \exp(-c_{\mathrm{pen}}\beta \sum_e \mathrm{price}(V_e Q_e(U)^{-1}))$ (9-01; c_{pen} is the penalty coefficient, `kernel.penalty` in the source's wording).

The push-forward value is linked to the coarse-step representation by the following three **identities** Source .

$$\mathrm{push}_\beta Y.\mathrm{eval} \overset{(a)}{=} \mathrm{gaussian}(Y) \overset{(b)}{=} \mathrm{earlyValue}(X, v, \beta, \tau, \alpha) \overset{(c)}{=} F.\mathrm{eval}. \quad (1)$$

a. `Return.earnings` (`YM/AnalyticBridge/VERIFIED_RESET_STEP/Inputs.lean:56`): the push-forward equals sector sum × sum over choices × prefactor × Haar integral over the frozen links × Gaussian integral (`gaussian`, :40).

b. `Return.atlas` (:70): a **proposition** that the Gaussian sector formula equals the "early representation" written with a list of activities X and value v. The docstring says it "does not prove which field shapes admit such a list". The instance of this proposition is built by `Rain.sprint` (`YM/PhysicalNext/RG_NEXT_COMPLETE_REPEATABLE_NONLINEAR_STEP/Rain/Glazing.lean:85`), and `Song.forward` (`Song/Embassy.lean:87`: early representation = push-forward) is its material.

c. `Collection.Local.actual` (`Evaluation.lean:93`): the early representation equals the value of the representation reread after extracting the marginal terms.

The **small-field one-step theorem** `Return.reset` (`Advance.lean:65`) Source takes the proposition (b), the ledger hypotheses and a continuous curve Y at step $m+1$, builds a

continuous curve F at step m, and states at each point: the licence condition (`licensed`), the one-step certificate `Move` (including the push-forward identity), for the coupling deviation $u = \beta F - \mathbf{predictor}$ the bounds $|u| \le \text{width}$ and $t^2|u| \le H$ (u equals the Δ of § 06 and corresponds to the correction d of 10-03), and the vacuum term $vF = \mathbf{vac}$. The one-step theorem `Use.Machine.run` (`Downstream.lean:65`) calls it, and `Mallow.Spring.moves` (`Mallow/Pediment.lean:58`) passes it the three inputs (ledger `Rain.varnish`, list `Rain.sprint`, continuity `Rain.blinking`) Source .

Which target it enters. The road to the existence theorem, `rodB` (`YM/FullVolume/Stage/Rods.lean:26`) → `Roof.Lyrics.supplier` (`YM/FullVolume/C/Songbook.lean:76`) → `Chord.spores` (:53) → `Spring.moves` → `Machine.run` → `reset`, has been checked by tracing the declarations literally Source . However, the witness in the verification record for `reset`, `Machine.run`, `Spring.moves`, `atlas` and `earnings` alike is `SharpGram.habitat` (the witness is only the first one found depth-first). The proof of `supplier` receives the one-step Move returned by `Spring.moves` and packs it into the step record. The lower-level lemmas of this chapter (`tree_gap`, `fog`, `late_cubic` and so on) have `SharpGram.habitat` as their witness in the record, and we have not checked whether they lie on the chain of the existence theorem.

02

Coordinates — tree gauge and relative coordinates

1. **Background.** Fix the coarse configuration V and place the background $\mathbf{replica}(V)$ on the fine lattice (`YM/PCalc/Chart.lean:35`). Links inside a block are 1, and links crossing blocks take the value of V Source .
2. **Tree gauge.** Span a tree in each block and fix the links on the tree. The gauge freedom is divided out by gauge transformations with the root fixed Source .
3. **Relative coordinates.** Free links are written $u_i = \exp(a_i)\,\mathbf{replica}(V)_i$ (a_i Lie-algebra valued) (`lifting`, :60) Source .
4. **Frozen links and moving links.** Links touching the neighbourhood of the seeds (radius $10R$) are left with the Haar measure (`Frozen`, `YM/SectorNext/GAUSSIAN_SECTOR_CHANGE/Shell.lean:42`), and only the remaining links are set as $a = \bar{a} + \hat{\beta}^{-1/2} z$ and integrated over Gaussian variables Source . The split is determined region by region (by seed type) and is not carried across steps.

03

The Gaussian part — precision, gap, windowed approximate inverse, decay

The quadratic part. For the coordinates a of the moving links, the action is split into a quadratic part $\frac{1}{2}|Da+h|^2$ and a remainder residual (`Remainder.lean:58`). D (`move`, `Affine.lean:208`) has rows for the linearised curvature of fine plaquettes and rows for the linearised penalty of coarse links. The precision is $\mathrm{gram} = D^{\mathsf{T}} D$ Source . On a flat background the precision has the form $K + c_{\mathrm{pen}}\,\Pi^{\mathsf{T}}\Pi$ (K is the kinetic part of the plaquette field, Π the linearisation of the predictor) (`FreeResolvent.baseMatrix`, `YM/Push/FREE_RESOLVENT/Flat.lean:81`) Source .

The gap on the tree section `tree_gap` (`Flat.lean:198`) Source : if the curvature part has gap a, then the gap of the flat precision restricted to the tree section is

$$\mathrm{gap} \;\geq\; \frac{\min(a,\,c_{\mathrm{pen}})}{1+\beta_L+\gamma_L} \tag{2}$$

at least this. β_L depends only on the block side L, the number of convolutions and the choice of kernel, and γ_L only on L; neither depends on the volume period. The penalty coefficient c_{pen} enters directly.

Decay `resolve_point` (`YM/Push/FREE_RESOLVENT/Inverse.lean:120`) Source : from a weighted row sum $\sum_j e^{\sigma d(i,j)}|A_{ij}| \leq M$ and a gap $r>0$, for $0 \leq \gamma \leq \min(\sigma/2,\; r\sigma/(4(M+1)))$,

$$|(A+u)^{-1}_{ij}| \;\leq\; e^{-\gamma d(i,j)}\,(r/2+u)^{-1}. \tag{3}$$

This is a Combes–Thomas-type estimate, with constants independent of both dimension and volume.

The windowed approximate inverse. The actual integral does not use gram^{-1}; instead the Gram matrix is inverted in a window around each address (pairs whose parent distance is within $R/20$), and the columns are collected and symmetrised into C (`fog`, `Chosen.lean:69`), which is used as the covariance Source . `SectorChange.windows` (`Windows.lean:184`) Source : for a symmetric A with a gap and a weighted row sum, there are $v, C', \lambda > 0$ such that, if the window radius ℓ is sufficiently large,

$$\|\mathrm{clouded}(W,A)-A^{-1}\|_{\text{unweighted row sum}}\le C'e^{-v\ell},\qquad \mathrm{clouded}(W,A)\text{ is positive definite with gap }\lambda$$

The difference is measured by the **unweighted** row sum. It is not claimed to be positive definite for an arbitrary window (the docstring says so explicitly).

Change of variables and where the error goes. Around the centre $\bar a$ (`meanLog`, `Chosen.lean:72`: the minimiser of the quadratic part approximated with the windowed inverse), set $a=\bar a+\hat\beta^{-1/2}z$ and integrate z against the Gaussian with covariance C. The integrand (`dyed`, `Change.lean:63`) contains $e^{-\beta\,\mathrm{residual}}$, the likelihood, the region weight, and the **error of the approximation** $\mathrm{adjustment}=\frac12(\cdots)-\mathrm{guide}(\bar a+gz)/g^2$ (`Remainder.lean:124`: the quadratic form of $C^{-1}-\mathrm{gram}$, the shift of the centre, constants) Source . Because the approximation is not taken outside but kept entirely in the integrand, (a) remains an identity. Because the covariance has finite range, Gaussian expectations over separated sets are exactly independent, which is the foundation of the later expansion Inferred .

04

The stationary point — agreement with the coarse action is carried as a condition

Stationary points appear in two ways Source .

1. **The centre of the integral** $\bar a$ (§ 03). A local approximation of the minimiser of the quadratic part.
2. **The stationary point of the combined action** ρ: a map $\phi\mapsto\rho(\phi)$ from the coarse logarithmic field ϕ to the fine moving coordinates, giving the stationary point of the combined action $\mathrm{tested}(\phi,\cdot)$ = (main action at step $m+1$) + $c_{\mathrm{pen}}\cdot$ (penalty). Its properties are written as four conditions (`YM/PCalc/Chart.lean`):
   - `Critical` (:88): $\rho(0)=0$, ρ is real-analytic, and near ϕ, $\rho(\phi)$ is a stationary point.
   - `PriceGap` (:100): the Hessian at 0 is $\ge\eta$.
   - `Guessed` (:97): the first derivative of ρ equals an explicit linear map.
   - `ClassicalMatch` (:108): for every choice of predictor i and $j\le4$, the j-th order jets **at the flat configuration** $\phi=0$ agree: $\phi\mapsto\mathrm{tested}_i(\phi,\rho(\phi))$ and $\phi\mapsto\mathrm{mainTerm}\ \mathrm{coarse\ path\ data}(\phi)$.

This agreement is a condition. The four conditions are **carried in the form of hypotheses** as fields of `Kin.Planned` (`YM/PCalc/Loop.lean:93`) (`critical`, `gap`,

`classicalPart` and so on), via the field `support` of the one-step theorem bundle `Directed` (`YM/PCalc/Band.lean:34`) and the field `paths` of the supply bundle (`YM/FullVolume/Roll/Fiscal.lean:37`), and `sailsW` (`Fiscal.lean:97`) closes them Source . The proof of `sailsW` uses `Stationary.verdant` and `Stationary.pouring`, and its hypotheses are only the simplicity of the group, the kernel's three obligations, the draft order and the sequence of constants. In the record, `sailsW` is reached from `continuum_with_position_tests` (called by the existence theorem) Source .

The main action of the coarse step is **path data given separately** (field `layer` of the supply bundle). Rather than building the coarse action from the stationary value, the form is to carry the condition that the given coarse action and the jet of the stationary value agree up to fourth order, and a separate theorem satisfies it Source . The action on the fine side is the main action at step $m + 1$, and it agrees with the Wilson action only at step N (field `peak` of `Planned`: agreement up to fourth order at step N; field `top` of the supply bundle `Imperial` (`YM/FullVolume/Roll/Fiscal.lean:38`): the path data at step N are those of the first step, 11-02) Source . This is a different object from the predictor (the explicit map that defines the kernel, 9-04).

05

Expansion of corrections, and the early representation

After the Gaussian integral, what remains is a product of local factors integrated against a finite-range covariance. We read the components of the expansion as hard-core Ursell coefficients and the Penrose-type tree inequality (`YM/Push/ABSOLUTE_SUMS`), Gaussian moments and cumulants of local words (`YM/Primitives/CONNECTED_GAUSSIAN_CUMULANTS`), and jets of connected activities (`YM/AnalyticNext/CONNECTED_ACTIVITY_JETS_E6`) Inferred (from names and docstrings).

The result is collected into a single dictionary and becomes the **identity** of `Song.forward`, "early representation = push-forward" Source . The early ledger `Song.loaded` (`Embassy.lean:108`) bounds the pieces of regular activities to the order of $\mathrm{octaveNeed}$ (`Song/Litho.lean:60`: the sum of $|\alpha|t^2$, β·cubic atoms, β·quartic atoms and one-loop atoms). Its size is $\leq \beta^{-7/16}$, larger than the exit ledger (`suns`, `Boulevard/Dawn.lean:76`) Source . After the same list is reorganised, it enters the exit ledger $\mathrm{forecastE} = \mathrm{credit} \cdot \hat{g}^{11/5}$ (`Rain.varnish`, `Glazing.lean:56`). Pieces with large support, multiplied by the weight $e^{-(w/4)B}$, are at most $\beta_f^{-23/20}$ (the third conclusion of `suns`, `Dawn.lean:76`; the bounding quantity is `gold`, :30) Source .

06

Extracting the marginal terms

The early representation is rearranged as follows (`Local.actual`, `reset`) Source .

1. **Vacuum term**: collecting the flat values of each regular-activity piece gives $v = \tau - \sum_i E_i(1)/|\Lambda|$ (`vac`, `Evaluation.lean:29`).
2. **Shift of the coupling**: collecting the log-Hessians of each piece in the curvature direction at the anchor gives, by hypercubic symmetry, the form $\delta_{ab}\Delta$ (`Averaging.isotropic`, `Isotropy.lean:77`). Δ (`delta`, :75) is moved into the coefficient of the kinetic term, $\beta_{\text{out}} = \beta_{\text{in}} + \alpha + \Delta$. α is the one-loop coefficient, held in advance by the path-data side.
3. **Normalising regular activities**: each piece is replaced by $\text{rebate} = (E - E(1)) - (\text{quadratic polynomial at the anchor})$ (`Residual.lean:28`). Regular activities are then 0 at the flat configuration with Hessian 0 in the affine directions, and return to the form of the condition `RegularFamily` of 11-01 § 03.
4. **How the coupling is fixed**: `reset` solves for β so that the density ratio matches at two test configurations (the flat configuration and a small-field configuration with no flags), and shows that the solution equals "predictor $+\Delta$" (`yield_shape`, `Advance.lean:25`). `yield_shape` says that the left-hand side of the rereading equation ((2) of 10-03) equals a positive constant multiple of $d - \Delta$, so the root is $d = \Delta$. This Δ corresponds to the correction d of 10-03, and the u of § 01 also equals Δ (proof of `reset`, `Advance.lean:54, :84`) Source . The coupling is fixed as the coefficient of the kinetic term in the representation (10-03).

07

The ledger — why $\hat{g}^{11/5}$

The regular-activity ledger is measured by "the supremum of finite-order derivatives in directions of size t", so a term of degree k in the field, $c \cdot (\text{field})^k$, contributes about $c\,t^k$. The sizes of what enters the regular activities can be read as follows Inferred (the supporting statements are Source).

What enters	Size	Basis
Fifth-order and higher remainder of the tree level	$\beta t^5 \approx \hat{g}^3 P^5$	The condition of jet agreement up to fourth order
Third-order and higher remainder of one loop	$t^3 \approx \hat{g}^3 P^3$	`FiniteBound` (`YM/PCalc/Loop.lean:62`) matches second derivatives
Cubic remainder of α	$(\log L)\, t^3$	`late_cubic`, `CubicGate` (`YM/UVNext/COUPLING_RESET/Precision.lean:71`, :96)

What enters	Size	Basis
Pieces with large support	$e^{-(w/4)B}$. $\mathrm{octaveNeed} \le \beta_f^{-23/20}$	`suns`, `gold`

The small pieces (first three rows) are of order $\hat{g}^{3-\varepsilon}$, and the pieces with large support of order $\hat{g}^{2.3}$ ($\beta_f^{-23/20}$); all fit within the $\hat{g}^{11/5}$ ledger. Meanwhile, the quadratic (marginal) coefficients are not left in the regular activities but moved to β (Δ and α), and the constants to v. $11/5$ can be read as a number between the marginal degree 2 and the first non-marginal degree 3, leaving room on both sides to absorb polylogarithms Inferred . `late_cubic` (:71) says that for any C, for sufficiently large x, $C\theta^3 \le \hat{g}^{11/5}$ ($\theta = \hat{g}P$) Source . The loss from rereading is prepaid by the step constant $\mathrm{credit} = 1/(64(1+\mathrm{base}))$ (`Tables.lean:36`) Source .

08

What this book's study had seen

This book has one step of the free field in Lean: the push-forward covariance $CTC^\dagger$ of the linearised block map, an upper bound on the covariance of the plaquette field, and a contraction uniform in the step (15-12, 15-13). The non-Gaussian step (vertex estimates, localisation, the loss of margin in hand-over) exists only as a paper study.

- What corresponds to this book's "linearised block map" is the linearisation Π of the predictor. The gap (2) and the decay (3) of the flat precision $K + c_{\mathrm{pen}}\Pi^{\mathsf{T}}\Pi$ correspond to this book's upper bound on the covariance of the plaquette field Inferred .
- The published construction has no structure that decomposes the covariance across steps and uses it (this book's contraction uniform in the step). It rebuilds the finite-range covariance of each step at each step Inferred .
- Replacing the covariance by a windowed approximate inverse and keeping the error in the integrand carries this book's "loss of margin in hand-over" as a term of the integrand rather than as a loss in the estimate Inferred .
- By redefining the ledger at every step as a function of that step's $\hat{\beta}$ and measuring it with a norm of finite-order derivatives, the construction makes the induction hypothesis not carry the losses in threshold and radius that accumulated step by step in this book's paper study Inferred .

09

What has not been read

- The content of the reorganisation from the early ledger to the exit ledger (`Marlin`, `Trio`, `Meadow`, `innings` of `Rain`), especially where the β·cubic atoms cancel.
- The content of the proof supplying `Kin.Planned` (from `YM/Next/STATIONARY` to `sailsW`).
- The Gram gap on non-flat backgrounds (`YM/SectorNext/GAUSSIAN_SECTOR_CHANGE/Gap.lean`) and how it connects to (2).
- The specifics of the type of the expansion (the dictionary of connected activities, the estimate of Ursell coefficients).
- The estimate of the normalising determinant (the content of τ in the vacuum term).
- The relation between the linearisation of the predictor and the kinetic term (transported curvature) of the coarse path data.

What this chapter can say

Can say	Cannot say
Source The one-step representation is linked by three identities: push-forward = Gaussian sector formula = early representation = reread representation (the instance of the list is `Rain.sprint`)	That the estimates use no approximation (the ledgers are inequalities)
Source Gap on the tree section (2), decay (3), error of the windowed approximate inverse (4) (unweighted row sum)	Positive definiteness for an arbitrary window
Source Agreement of jets up to fourth order at the flat configuration between the stationary value and the coarse action is carried as a condition of `Kin.Planned`, supplied by `sailsW`	Building the coarse action from the stationary value (the coarse action is path data given separately)
Source Marginal terms: flat values go to the vacuum term, the quadratic at the anchor Δ to the coupling, and the rest is normalised by rebate	The content of the reorganisation lemmas
Inferred The ledger $\hat{g}^{11/5}$ is a number between the marginal degree 2 and the first non-marginal degree 3	That the reason for choosing $11/5$ is written in the source

Sources and reproduction

Item	Kind	Source / tool
Three identities and the one-step theorem	Source	`YM/AnalyticBridge/VERIFIED_RESET_STEP/Inputs.lean:40`, :56, :70, `Evaluation.lean:29`, :93, `Advance.lean:25`, :65, `Downstream.lean:65`
Instance of the list and its material	Source	`YM/PhysicalNext/RG_NEXT_COMPLETE_REPEATABLE_NONLINEAR_STEP/Rain/Glazing.lean:56` :85, `Song/Embassy.lean:87`, :108, `Song/Litho.lean:60`, `Boulevard/Dawn.lean:30`, :76 `Mallow/Pediment.lean:58`
Road to the existence theorem	Source	`YM/FullVolume/Stage/Rods.lean:26`, `YM/FullVolume/C/Songbook.lean:53`, :76
Coordinates	Source	`YM/PCalc/Chart.lean:35`, :60, `YM/SectorNext/GAUSSIAN_SECTOR_CHANGE/Shell.lean:42`
Gaussian part	Source	`YM/Push/FREE_RESOLVENT/Flat.lean:81`, :198, `Inverse.lean:120`, `YM/SectorNext/GAUSSIAN_SECTOR_CHANGE/Affine.lean:208`, `Remainder.lean:58`, :124, `Windows.lean:184`, `Chosen.lean:69`, :72, `Change.lean:63`
Conditions on the stationary point and their supply	Source	`YM/PCalc/Chart.lean:88`, :97, :100, :108, `YM/PCalc/Loop.lean:62`, :74, :93, `YM/PCalc/Band.lean:34`, `YM/FullVolume/Roll/Fiscal.lean:37`, :97
Marginal terms and the ledger	Source	`YM/AnalyticBridge/VERIFIED_RESET_STEP/Isotropy.lean:75`, :77, `Residual.lean:28`, `Tables.lean:36`, `YM/UVNext/COUPLING_RESET/Precision.lean:71`, :96

Words used in this chapter — relative coordinates, tree gauge, precision, gap on the tree section, windowed approximate inverse, stationary point, jet agreement, early representation, vacuum term, shift of the coupling. Their meanings are in the A-02 glossary.

Next chapter to read: 11-10 Convergence at a fixed step, and the step that removes the step index — the argument that holds the step fixed and lets $N \to \infty$.

Convergence at a fixed step, and the step that removes the step index

> This chapter is established knowledge (a reading of the published construction). § 06 is this book's own investigation and § 07 is a map of open questions.

What this chapter shows — Why the ingredients of the representation at step m (coupling, activities, path data) converge along the whole sequence in the cutoff N. The source is not "an estimate comparing N and N' quantitatively" but compactness, uniqueness by anchoring at step 0 and contracting from above, and the subsequence principle. From there, via uniform convergence of densities, images of observables, and the step that removes the step index, the ultraviolet limit at fixed volume comes out along the whole sequence.

Chapters assumed — 11-01 (representation), 10-04 The shooting method ($\beta = \bar{\beta}$ at step 0), 8-05 Correlation functions and the order of limits, 8-04 Renormalised insertions.

Said first — The convergence is qualitative; no rate is in the statements. It is convergence for each volume p, with no statement uniform in p. The limiting objects (the limiting measure, the limiting representation) are not defined. This book has not read the source of the one-step map's contraction constant $1/16$ (the analytic estimates of the one-step map).

The order of this chapter

1. Statement — convergence of ingredients, uniform convergence of densities, the estimate that removes the step index, the ultraviolet limit
2. Outline of the mechanism 1 — the source of convergence of ingredients
3. Outline of the mechanism 2 — densities, observables, calibration
4. Outline of the mechanism 3 — the step that removes the step index
5. The limiting objects are not constructed — what can and cannot be said about the step-0 theory.
6. What this book's study had seen
7. What has not been read

Labels: Source = read in the source of the published construction / Inferred = this book's inference / Assessment = according to the committee's assessment. The other labels (Proved here, Cited, Reading, Lean and so on) mean the same as in the rest of the book (for details see "Labels" in 6-00).

01

Statement — convergence of ingredients, uniform convergence of densities, the estimate that removes the step index, the ultraviolet limit

The ultraviolet limit at fixed volume (Theorem 4.1 of the committee's assessment) Assessment : the construction chooses a sequence of spacings $a_N \to 0$, a sequence of volumes L_p and couplings $\beta_{N,p}$, and for each volume p, each sequence of labels and each test function (avoiding coincident points, with support inside the volume), the limit $S_p(v;f) = \lim_{N\to\infty} C_{N,p}(v;f)$ of the correlation $C_{N,p}(v;f)$ exists **along the whole chosen sequence of spacings**, and is represented by a C^∞ kernel away from coincident points (assessment § 4.3). It also states that there is no universality with respect to the sequence of spacings Assessment . On the Lean side this is `Campaign.realUV : PositionsUV R.family Sequence.full` (`YM/FullVolume/Fields/Filled.lean:45`) Source . The public statement, too, requires convergence along the whole sequence in the cutoff N for each volume p; only the sequence of volumes is a subsequence (`Converges`, `PublicWilson.lean:85-90`) Source . The strong form with kernels and differences (`PositionsUV … Sequence.full`) appears only in the conclusion of the construction theorem `constructed` Source .

The components of that line are as follows Source . All of them are reached in the record from `continuum_with_position_tests` (called by the existence theorem).

Order	Statement	Location
1	Bundle of convergence of ingredients: from step k up, $\beta(N,p,i)$ is Cauchy, the activities are Cauchy in the norm `Show`, and the path data converge	`Helmet` (`YM/FullVolume/Gables/Honors.lean:30`), supply `helmed` (:62)
2	Source: a sequence of trajectories whose step-0 couplings converge has ingredients at all steps converging along the whole sequence	`Window.allB` (`YM/FullVolume/Glide/Triumph.lean:37`)
3	The effective density at step m (with the vacuum constant removed) is uniformly Cauchy in N in the sup norm over configuration space, and so is its total mass	`Flux.fanned` (`YM/FullVolume/Fields/Cement.lean:50`), `Flux.lacks` (:56)

Order	Statement	Location
4	Images of observables at step m are uniformly Cauchy	`Rising.hail` (`YM/FullVolume/Fields/Capital.lean:57`)
5	Difference between the true lattice correlation and the readout at step $j \le \mathrm{refreeze}/(e^{Bj/64}/2)$ (independent of N)	`Sever.echoes` (`YM/FullVolume/Fields/Peat.lean:42`)
6	The true lattice correlation is Cauchy in N	`Rising.arrives` (`YM/FullVolume/Fields/Timber.lean:74`)
7	Uniform boundedness + pointwise Cauchy at general positions ⇒ locally uniform convergence with derivatives	`positioned` (`YM/FullVolume/Profiles.lean:42`)

"Uniformly Cauchy" is the type `Confluence` (`YM/FullVolume/Fields/Basin.lean:10`), consisting of eventual uniform boundedness and $\forall \varepsilon\ \exists N_0\ \forall n, n' \ge N_0\ \forall V :\ \mathrm{dist}(f_n(V), f_{n'}(V)) \le \varepsilon$ Source . The hypothesis `Flux` of item 3 is given by `Rising.flooded` (`Capital.lean:31`) at steps $k \le m$, and in the main theorem $k = 0$ (`sees`, `Filled.lean:30`). So it holds for every step $m \ge 0$, every p, along the whole sequence of cutoffs Source .

02 Outline of the mechanism 1 — the source of convergence of ingredients

Coordinates. The coupling at step m is measured in the ratio coordinate λ_m relative to the reference value $\hat{\beta}_m$ (the allowed band is a bounded interval), and the activities in the norm `Show` on the scale of step m (`YM/FullVolume/Table/Tabs.lean:66`). `Show` is the supremum, over points, of the sum over anchored polygons of the suprema of derivatives weighted by an exponential of the covering number, divided by the step's ledgers ($\hat{g}_m^{11/5}$ and $e^{-P_m^{1/5}}$); it is independent of the volume Source . On every trajectory the norm of the activities is at most $1/8$ Source .

Contraction of the one-step map `Muse.ticket` (`YM/FullVolume/Gain/Rate.lean:94`) Source : with the max norm on (λ, K),

$$\|\Delta K_m\| \le \tfrac{1}{16} \max\big(|\Delta\lambda_{m+1}|, \|\Delta K_{m+1}\|\big), \qquad |\Delta g_m| \le 8 f_m \max(\cdots). \quad (1)$$

g_m is the one-step deviation of the coupling (the remainder after removing the one-loop part), and f_m is a step-dependent constant. The proof is a Cauchy estimate from analyticity Source (statement read) / Inferred (reading of its meaning). `Window.curls` (`Glide/Duty.lean:45`) writes the same estimate in the metric of the completion.

Four components Source .

1. **Compactness** (`thinning`, `Triumph.lean:28`): λ_m at each step lies in a bounded interval, so by compactness of a countable product a subsequence converging simultaneously at all steps can be extracted.
2. **If the couplings converge, so do the activities** (`bankA`, :25): the difference from the trajectory restarted with activities 0 at step b is at most $(1/16)^{b-m}$ at step m (`Window.culprit`, `Glide/Wings.lean:59`, forgetting of the top end). Restarted trajectories are compared with each other by continuity over finitely many steps. It closes with $\varepsilon/3$.
3. **Uniqueness** (`Window.accord`, `Glide/Coincide.lean:58`): two limiting trajectories whose step-0 coupling limits are equal agree at all steps.
4. **Subsequence principle** (`travels`, `YM/FullVolume/Artes/Events.lean:11`): if every subsequence has a further subsequence converging to the same limit, the whole sequence converges.

The self-estimate for uniqueness. Let the difference at step i between two trajectories be $y_i = \max(|\Delta\lambda_i|, \|\Delta K_i\|)$, and $Y = \sup_i y_i$. Activities contract from above (the fine side), giving $\|\Delta K_i\| \le Y/16$. The couplings are equal at step 0 (the anchor), and propagate towards the fine side by the sum of one-step deviations (`arrest`, `Coincide.lean:43`):

$$|\Delta\beta_i| \le \sum_{j<i} 4\,\mathrm{stow}(j)\,Y \le 4Y \cdot \mathrm{glint} \cdot \mathrm{brace}(i). \tag{2}$$

The last inequality is the calendar condition `Search.radiusB` (`YM/FullVolume/Gain/Motion.lean:48`: $\sum_{i<m} \mathrm{stow}(i) \le \mathrm{glint} \cdot \mathrm{brace}(m)$). Changing coordinates gives $|\Delta\lambda_i| \le Y/32$. Together, $Y \le Y/16$, hence $Y = 0$ Source .

Meaning Inferred : this is uniqueness of the solution of a two-point boundary value problem on steps $[0, \infty)$, with the coupling at step 0 and "bounded activities" at step ∞ as boundary conditions. The marginal direction (the coupling) is anchored at step 0, and shooting (10-04) aligns it to $\beta = \bar\beta$ for all N, p. The irrelevant directions (the activities) contract from above. The difference at the top end (the bare Wilson step) need not be small; being bounded is enough.

The chain in the source is `helmed` ← `Winter.convergence` ← `allB` (`Winter`, `Glide/Vac.lean:31`) Source . No rate comes out, because it goes through compactness Source .

03

Outline of the mechanism 2 — densities, observables, calibration

Densities. The representation at step m is a continuous function of its ingredients: the path action is uniformly continuous in the coupling and the path data, the sum of activities is continuous, and the gas is a finite sum and product over boxes. Since `Confluence` is closed under sums, products and composition with continuous maps, the density is uniformly Cauchy in the sup norm (`fanned`), and since configuration space is compact and the reference measure is a probability measure, the total mass is Cauchy too (`lacks`) Source (statements read) / Inferred (reading of the line).

Uniform convergence of the normalised density $\mathbf{eval}/Z_N$ is not stated as a named statement. It follows in one line from `fanned`, `lacks` and a positive lower bound on the denominator independent of N (`lingers`, `YM/FullVolume/Fields/Till.lean:20`). Convergence in sup norm includes convergence in total variation Inferred.

Calibration of observables. The coefficients of local fields of degree $\le d$ live in a finite-dimensional space; one step maps them by a linear isomorphism `leanings` (`Club` in `YM/FullVolume/Mark/Nursery.lean:11`) whose inverse is uniformly bounded, and the remainder contracts by $1/2$ per step. The coefficients are solved backwards so that reading at the reference step $r = \mathbf{ref}(d)$ gives exactly v (8-04), and carrying them from step r by the inverse maps makes the coefficients at steps $m \ge r$ Cauchy in N (`Gear.spends`, `YM/FullVolume/Mark/Sages.lean:66`; `Branch.winds`, `stays`, `YM/FullVolume/Gables/Master.lean:44`, :50) Source. The structure is the same "anchored marginal direction + contracting remainder" as in § 02, and since it is linear and finite-dimensional, no compactness is needed Inferred.

04

Outline of the mechanism 3 — the step that removes the step index

With step j fixed, the readout on the representation at step j using the step-j images of observables is Cauchy in N (`latter`, `Timber.lean:58`) Source. On the other hand, `echoes` states

$$\left\| \text{true lattice correlation} - \text{readout at step } j \right\| \;\le\; \frac{\mathrm{refreeze}}{e^{B_j/64}/2}, \qquad B_j = \lceil (\log \hat{\beta}_j)^2 \rceil \tag{3}$$

under the hypothesis (`Drake`) that the separation of points is at least B_j^2 in units of step j Source. The right-hand side does not depend on N. refreeze

(`YM/FullVolume/Fields/Flight.lean:28`) contains an exponential of the number of lattice points at the reference step and is not uniform in the volume p Source . The smallness $e^{-B_j/64}$ is slower than any power of a_j Inferred . In physical length $B_j^2 a_j \to 0$, so fixed separated points satisfy the hypothesis at sufficiently fine steps Inferred .

`arrives` takes $\varepsilon/4$, chooses a step j at which the right-hand side of (3) is at most $\varepsilon/4$, and by `latter` and the triangle inequality shows that the true lattice correlation is Cauchy in N Source . Finally `positioned` extends from dense points to locally uniform convergence with derivatives, using uniform boundedness (`FixedBounds`) and equicontinuity Source .

Here is the picture in words. Put the step on the vertical axis (0 at the bottom, the fine side at the top) and the cutoff N on the horizontal axis; the trajectory for each N is a vertical line from step N (bare Wilson) down to step 0 (coupling $\bar{\beta}$). On a horizontal line at fixed height m, the cross-section (the representation at step m) converges as N grows: that is § 02 and § 03. `arrives` says that if the height j is taken high enough, the difference between the top (the true correlation) and the cross-section at height j is small independently of N. Step 0 is where the units are fixed; it is not used in estimating correlations.

05

The limiting objects are not constructed

- There are no named definitions of the limiting measure $\mu_{\infty,p,m}$ at step m, the limiting density, or the limiting representation Source (search scope: FullVolume, RG, Extraction). What exists goes only as far as convergence of the ingredients (coupling Cauchy, activities Cauchy, existence of some limiting path data).
- It is not stated that the limiting density has a representation of the same form, or that it is consistent across steps, $\mu_{\infty,p,m} = \mathrm{push}(\mu_{\infty,p,m+1})$ Source .
- Because the one-step kernel depends on $\beta(N, p, m+1)$, the push-forward measure at step m is not "the Wilson measure pushed forward by a fixed map"; it is an object whose map itself moves with N Source (the form of `Transfer`).
- The line of proof does not construct limiting objects; from Cauchy at a fixed step it removes the step index and goes directly to Cauchy for the true lattice correlations.
- Reflection positivity of the measures at renormalised steps (including step 0) is not stated Source (within the scope of the name search). The statement of reflection positivity is `Slice.reflected_positive` (`YM/Slice/Reflection.lean:98`) for the bare Wilson measure. Since the kernel's patches cross the plane, we read that the push-forward does not inherit it automatically Inferred .

- There is no renormalisation to scales coarser than step 0 (spacing 1). Step indices are natural numbers, and step 0 is the coarsest step Source (within the scope of the name search).
- Volume: the contraction and Lipschitz constants do not depend on p, but the convergence is a qualitative statement for each p. Estimates at the level of the density (`refreeze`, the lower bound on the denominator) include an exponential in the volume. The one-step ledgers (`EBound`, `KBound`, `Show`), on the other hand, are pointwise sums and do not depend on the volume Source .

What can and cannot be said about the step-0 theory

When the remaining question is restated as "the long-distance question for the effective lattice theory at step 0 (spacing 1)", here is what can and cannot be said from the source.

Item	In the source	Not in the source
Coupling	$\beta(N, p, 0) = \bar{\beta}$ for all N, p (`Selected.reaches`, `YM/RG/History.lean:239`) Source	—
Limiting objects	The effective density at step 0 is uniformly Cauchy in N (p fixed) Source	A definition of the limiting measure and its representation Source (within the scope of the name search)
Images of observables	Images at step m exist for observables with reference step $\psi(d) \leq m$ (hypothesis of `hail`, `YM/FullVolume/Fields/Capital.lean:57`) Source . At step 0, only for observables whose reference step is 0 or below	Images at step 0 of observables whose reference step is above 0
Volume	What is independent of the volume is the one-step ledgers (pointwise sums) Source	A form of the density-level estimates uniform in the volume (`refreeze` and others include an exponential in the volume)
Reflection positivity	For the bare Wilson measure (`Slice.reflected_positive`) Source	For the measure at step 0

Item	In the source	Not in the source
Entry for the one-step estimate	For a smoothing index j and a coupling value x, of the form "holds if $\mathrm{betaEntry}(j) \le x$" (`Delays.betaEntry`, `YM/Price/Intent.lean:116`; `Opened`, `YM/AnalyticBridge/VERIFIED_RESET_STEP/Opening.lean:45`) Source	A formula or number for the entry value. The value is extracted as an existence from a bundle of properties that "hold for sufficiently large x"
Reference coupling $\bar\beta$	$\bar\beta > 1$, at least the first entry (`Bar.ready`), and $x^{-1/2} e^{(\log\log x)^2} \le 1$ for all x at least $\bar\beta$ (`Bar.threshold`, `YM/Price/Intent.lean:126-127`) Source . From the last condition alone, $\bar\beta \gtrsim 9 \times 10^5$ Reading (root found numerically)	An explicit value of $\bar\beta$
Renormalisation towards the coarse side	—	Steps coarser than step 0 (steps flowing towards strong coupling)
Relation to coarse Wilson loops	— (no statement)	The relation between Wilson loops of step-0 configurations and Wilson loops of the fine field (or the continuum) Source (within the scope of the name search; 16-03 § 04)

06

What this book's study had seen

The ultraviolet chain of this book's study took **subsequential limits** of the family of push-forward measures $\nu_{n,m}$ on a fixed coarse lattice and transferred properties to them (28-01 Implication of subsequential limits).

- Convergence at a fixed step in the published construction sits in the position of this book's "subsequential limit", and it comes out without subsequences Source (statement read) / Inferred (reading of the correspondence).
- The objects are not the same, however. The push-forward of the published construction is a composition of probability kernels depending on $\beta(N, p, m+1)$; this book's push-forward is a deterministic block map independent of N. Convergence of this book's $\nu_{n,m}$ does not follow from the convergence in the published construction Inferred .

- Patterns that can be borrowed as they are Inferred : the skeleton that eliminates subsequences (compactness, uniqueness, subsequence principle), forgetting of the top end, and the self-estimate pattern $Y \leq Y/16$. None of these depends on whether the map is deterministic.
- What cannot be borrowed Inferred : the representation itself keeping the same form at every step (the same point as the handling of large fields), and the **anchor**. This book's design supplies β_N from outside, so the marginal direction has no anchor. The subsequence ambiguity in this book may come from here.

07

What has not been read

- The source of the analytic estimates of the one-step map (`Muse.income`, `Pass.income`). This is where the contraction constant $1/16$ comes from, and it is the body of the small-field and large-field analysis.
- The continuity comparing one-step maps with different cutoffs (`Pith.shall` inside `Bill.taxes`).
- Uniqueness of shooting at finite N (the content of the supply of `Jewel.unique`).
- The matrix form of the coefficient map `leanings` (whether mixing with the identity field and lower-degree fields is triangular).
- Completeness of `Show`, and existence of the limiting activities.
- The meaning of `Next.Free.Amplify` (field `rises` of `Helmet`).

What this chapter can say

Can say	Cannot say
Assessment The ultraviolet limit at fixed volume exists along the whole chosen sequence of spacings (Theorem 4.1 of the assessment). The public statement (`Converges`) also requires the whole sequence in the cutoff; only the volumes are a subsequence Source	Universality with respect to the sequence of spacings, convergence uniform in the volume, uniqueness of the infinite-volume limit
Source The source of convergence of ingredients is compactness + uniqueness by the step-0 anchor and contraction from above + the subsequence principle (`allB`)	The rate of convergence (not in the statements)
Source The effective density at step m is uniformly Cauchy in N in the sup norm (all $m \geq 0$, p, whole sequence)	A named statement of convergence of the normalised density (it follows in one line but is absent)

Can say	Cannot say
Source The difference from removing the step index is at most $\mathrm{refreeze} \cdot 2e^{-B_j/64}$, independent of N (point separation $\geq B_j^2$)	An estimate of this difference uniform in p
Source For the limiting measure, the limiting representation and consistency across steps there is neither definition nor statement	Properties of the limiting objects (reflection positivity and so on)
Source Reflection positivity is stated only for the bare Wilson measure, and there is no renormalisation coarser than step 0	Reflection positivity of the measures at renormalised steps (including step 0)

Sources and reproduction

Item	Kind	Source / tool
Ultraviolet limit at fixed volume	Assessment	Assessment § 4.3 (Theorem 4.1)
Convergence of ingredients and its source	Source	`YM/FullVolume/Gables/Honors.lean:30`, :62, `Glide/Vac.lean:31`, `Glide/Triumph.lean:25`, :28, :37, `Glide/Coincide.lean:43`, :58, `Glide/Wings.lean:59`, `Glide/Duty.lean:45`, `Gain/Rate.lean:94`, `Gain/Motion.lean:48`, `Artes/Events.lean:11`, `Table/Tabs.lean:66`
Densities, observables	Source	`YM/FullVolume/Fields/Basin.lean:10`, `Cement.lean:50`, :56, `Capital.lean:31`, :57, `Till.lean:20`, `Filled.lean:30`, :45
Calibration	Source	`YM/FullVolume/Mark/Nursery.lean:11`, `Mark/Sages.lean:66`, `Gables/Master.lean:44`, :50
The step that removes the step index	Source	`YM/FullVolume/Fields/Peat.lean:42`, `Flight.lean:28`, `Timber.lean:58`, :74, `YM/FullVolume/Profiles.lean:42`
Dependency record	Source	`verification/declaration-usage.tsv` (the declarations of this chapter are used, witness `continuum_with_position_tests`)

Words used in this chapter — convergence of ingredients, uniformly Cauchy, forgetting of the top end, anchor, two-point boundary value problem, subsequence principle, calibration, the step that removes the step index. Their meanings are in the A-02 glossary.

Next chapter to read: 11-11 What this part has not yet read — the list for the whole part. After that, on to Part 12 Infinite volume and reconstruction.

What this part has not yet read

This chapter is a map of open questions. § 01 is guide material.

What this chapter shows — The "what has not been read" items written in each chapter of Part 11, collected into one table, with their locations (relative file paths and declarations) and which claim of the part each one balances.

Chapters assumed — 11-00 (labels and levels of citation). The subject of each row is in the chapter in the right-hand column of the table.

Said first — What is listed here are places where this book has not read the source. It does not mean there are gaps in the proofs of the published construction. The results of the published construction remain at the level at which the committee assessed them as proved in Lean.

The order of this chapter

1. How to read this list
2. The large rows — places that balance the part's claims
3. List by chapter
4. Declarations whose target has not been checked
5. Cautions for reading

Labels: Source = read in the source of the published construction / Inferred = this book's inference / Assessment = according to the committee's assessment. The other labels (Proved here, Cited, Reading, Lean and so on) mean the same as in the rest of the book (for details see "Labels" in 6-00).

01

How to read this list

Each row gives what has not been read, its location (path relative to `YM/` and the declaration name), and which claim in the part that place bears on. The locations were checked by this book in the source as far as names and types. The content of the proofs has not been read. Where a location is only a candidate, it is marked "candidate".

02

The large rows — places that balance the part's claims

Not read	Location	Claim it be
Where the Haar-loss lemma is. Which lemma puts, and where, the loss relative to Gaussian normalisation for links integrated with the Haar measure near the seeds. `Marginal` (45 declarations), `Haar` (3), `Leases` (26) and `Grant` (10) in the large-field location are all outside the dependencies	Candidates: `YM/SectorNext/GAUSSIAN_SECTOR_CHANGE/Haar.lean`, `Counting.lean`, `YM/AnalyticBridge/SELECTED_ORDERED_KERNELS`, `YM/AnalyticNext/CONNECTED_ACTIVITY_JETS_E6/IssuePlan.lean:34` (`Expense`)	The readi that it ent the field f logarithm cost per s (11-05 § 0

Not read	Location	Claim it be
The whole of the upper bounds on the remaining integrals after the price is extracted. `peel` does not say how large the remaining (outgoing) integral is, and the docstring of its location also says "conditional on the cost of the exit multiplier"	`YM/Next/FLUCTUATIONS/Sector.lean:229` (`peel`), `YM/Push/ABSOLUTE_SUMS/Grants.lean` (docstring), `YM/SectorNext/LARGE_FIELD_REMAINDERS` (`Annulus.lean:47`, `CorePacket.lean:62`) → `YM/AnalyticBridge/ANNULUS_RATIO_INSERTIONS` → `YM/PhysicalNext/V9_JOINT_ENDPOINT_LOAD`	That the p seed facto actually reaches th one-step e (11-04 § 0.
The content of how the one-step theorem is assembled. Where the Move is produced from the three inputs (identity of the list, ledger, continuity), and how it ties to the large-field ledger	`YM/AnalyticBridge/VERIFIED_RESET_STEP/Downstream.lean:65` (`Use.Machine.run`), `YM/PhysicalNext/RG_NEXT_COMPLETE_REPEATABLE_NONLINEAR_STEP/Venue.lean:47` (field `machine`)	The one-s certificate 00 § 03)
The reorganisation lemmas. The reorganisation from the early ledger $\beta^{-7/16}$ to the exit ledger $\hat{g}^{11/5}$ (cancellation of the cubic atoms), and the lemmas inside the part that builds the instance of the list	`YM/PhysicalNext/RG_NEXT_COMPLETE_REPEATABLE_NONLINEAR_STEP/Marlin` (`carries` and others), `Trio`, `Meadow`, `Beam` (`travels`), `Rain` (`innings`)	The regul activity le (11-09 § 0.

Not read	Location	Claim it be
The whole construction that rebuilds new forced activities after one step and fits them into the ledger	`YM/PhysicalNext/RG_NEXT_COMPLETE_REPEATABLE_NONLINEAR_STEP` (`Roost`, `Lotus`, `Sward`), `YM/AnalyticNext/CONNECTED_ACTIVITY_JETS_E6` (from `ratesCombine` in `OptionCost.lean:39` onward)	The balan of the rea of persist across ste (11-07 § 0
The source of the analytic estimates of the one-step map. Where the contraction constant 1/16 comes from	`YM/FullVolume/Gain/Rate.lean` (`Muse.income`), `Pass.income`	Converge ingredien 10 § 02)

03

List by chapter

Chapter	Not read	Location
11-01	The meaning of all the fields of `Bounds` for the path data	`YM/RG/PathActions.lean:173`
11-01	The role of the additional condition on forced activities (`extraTest`)	`YM/RG/Activities.lean` (`ForcedAddress`)
11-02	The construction that arranges the first forced activities into polymers, and their entry into the first ledger	`YM/Extra/WILSON_STARTUP/Packets.lean`, `Ink.lean`
11-02	The correspondence between due read in the orientation frame (`owed`) and due	`YM/Extra/WILSON_STARTUP/Ownership.lean:193`
11-03	Assembling the smooth partition of unity into a partition over seed types, and how the transition width is assigned	`YM/Push/SECTOR_MEASURE/Profile.lean` (`Chi`)

Chapter	Not read	Location
11-03	The description of overlaps between regions; how the blocking kernel Φ and the distribution to the noise are built	`YM/Push/SECTOR_MEASURE/Overlap.lean`, `YM/Pressure/Rows.lean` (`leap`, `kick`)
11-04	The proofs of the three inequalities: coarse plaquettes, fine plaquettes and penalty, payment for large coarse plaquettes	`YM/Work/E05`–`YM/Work/E07`
11-04	The meaning of the tube fields (`Deep`, `Roll`) of the fitting condition `Funds`	`YM/Pressure/Seeds.lean:101`
11-05	The proof of the sum of polymers containing a seed (Ursell coefficients, Penrose-type tree inequality, dictionary of connected activities)	`YM/Push/ABSOLUTE_SUMS/Ursell.lean`, `Graph.lean`, `YM/AnalyticNext/CONNECTED_ACTIVITY_JETS_E6`
11-05	All the fields of the combinatorial condition `Buy`; the definition of the polymer-scale quantity	`YM/Push/ABSOLUTE_SUMS/Mixed.lean`, `Bulkhead.lean`
11-06	The map by which values of unflagged polymers enter the addresses of regular activities	`YM/AnalyticNext/CONNECTED_ACTIVITY_JETS_E6/Colony.lean` (from `entries` onward)
11-06	The handling of incoming activities (`Sales`, `ratesCombine`) and what follows	`YM/AnalyticNext/CONNECTED_ACTIVITY_JETS_E6/OptionCost.lean:39`
11-07	Where the strong ledger of the one-step certificate, `Move.forced`, is used at later steps	Unconfirmed

Chapter	Not read	Location
11-07	That action differences are not counted twice across steps (in the form including the difference between the orientation-averaged action and the action of a single orientation)	`Testing.ships` and others (unconfirmed)
11-08	The content of the price condition `Ringing`, the explicit form of the condition that $\bar{\beta}$ be taken large, the chain constructing a calendar that satisfies `Fits`	`YM/Pressure/Seeds.lean:155`, :161, `YM/Price/Intent.lean:123`
11-09	The content of the proof supplying the stationary-point conditions	`YM/Next/STATIONARY` → `YM/FullVolume/Roll/Fiscal.lean:97` (`sailsW`)
11-09	The Gram gap on non-flat backgrounds, and how it connects to the gap in the flat case	`YM/SectorNext/GAUSSIAN_SECTOR_CHANGE/Gap.lean`
11-09	The estimate of the normalising determinant (content of the vacuum term); the relation between the linearisation of the predictor and the kinetic term of the coarse path data	Unconfirmed
11-10	The continuity comparing one-step maps with different cutoffs; uniqueness of shooting at finite N	`YM/FullVolume/Glide/Bill.lean` (`Bill.taxes`), `YM/FullVolume/Finale/Program.lean:133` (`Jewel`)
11-10	The matrix form of the coefficient map (mixing with the identity field and lower-degree fields); completeness of `Show` and the limiting activities	`YM/FullVolume/Mark/Hinge.lean` (`Gram.reads`), `YM/FullVolume/Table/Tabs.lean:66`

What the source lacks about the step-0 theory

For details see the table in 11-10 § 05. Here we list only what is not in the source `Source` (within the scope of the name search).

- A definition of the limiting measure at step 0, and its representation.
- Images at step 0 of observables whose reference step is above 0 (`hail` assumes $\psi(d) \le m$).
- A form of the density-level estimates uniform in the volume (only the one-step ledgers are independent of the volume).
- Reflection positivity of the measures at renormalised steps (including step 0). Renormalisation coarser than step 0.
- The relation between Wilson loops of step-0 configurations and Wilson loops of the fine field (or the continuum) (relation to coarse Wilson loops; 16-03 § 04).
- Explicit values of the entry for the one-step estimate $\mathrm{betaEntry}(j)$ (`YM/Price/Intent.lean:116`) and of the reference coupling $\bar\beta$. The source contains only existence and the conditions `Bar.ready` and `Bar.threshold` (:126-127); from the latter alone, $\bar\beta \gtrsim 9 \times 10^{5}$ Reading .

04

Declarations whose target has not been checked

For declarations that are "used" in the verification record (in the dependencies of one of the 17 headline theorems) and whose witness in the record is `SharpGram.habitat`, this book has not checked whether they lie on the chain of the existence theorem. The witness in the record is only the first one found by depth-first search, so this does not mean they are not on it either Source (how the record is made). Those cited in this part are as follows.

- Large fields: `WilsonStartup.due_eq`, `owed_zero`, `Tube.locates`, `Stores.Plot`, `Allocation.flagged`, `Chi.share_sum`, `Bid.Crowds`, `Seals`, `Lapse`, `Teller.Grants`, `Sector.peel`.
- Prices and conversion: `Buy.quote`, `Cash.armK`, `dictionary_fit`, `eventual_resize`, `Time.late_force`, `Time.now`, `Postage.saved`, `Collection.vac`.
- Lower-level small-field lemmas: `tree_gap`, `resolve_point`, `late_cubic`, `Rain.sprint`, `Song.forward`. The road up to the one-step theorem `Return.reset`, however, has been traced literally from the existence-theorem side (11-09 § 01).

Checked to be outside the dependencies (outside): `Remainder.hole_price`, `bulk_price`, `Marginal`, `Haar`, `Leases` and `Grant` in the large-field location, `Allocation.hard`, `Directed.holding` Source .

05

Cautions for reading

- Many declaration names and file names in the source do not describe their content. Following words in the docstrings ("freeze", "hole", "delay" and so on) as clues can lead to reading paths outside the dependencies. It is safer to check in the verification record whether something is used before reading it.
- The source's docstrings contain many notes to the proof writer (of the form "do not ..."). This book treats them as cautions to the writer and does not cite them as mathematical claims.
- The equation numbers of the paper that the source cites (numbers for the row identity, the body of the truncated action and so on) could not be traced, because the paper has not been published.

What this chapter can say

Can say	Cannot say
Source The location of each row (file and declaration name) was checked in the source as far as names and types (except those marked as candidates)	The content of the proofs in each row
Source Declarations outside the dependencies (`hole_price` and others) are "outside" in the verification record	That paths outside the dependencies are mathematically wrong (nothing is said)
Assessment The dependencies of the 17 headline theorems were rechecked in a freshly initialised kernel environment (assessment §9.1)	That this book has checked that the places in this list contain no errors

Sources and reproduction

Item	Kind	Source / tool
Lean source of the published construction	Source	`yang-mills-group/OpenAI-YM-20261028` (Apache-2.0). Locations as in each row
Dependency record	Source	`verification/declaration-usage.tsv` (status, headline witness)
The committee's assessment	Assessment	`docs/assessment-1.pdf` in `yang-mills-group/YM` (2026-11-10), §9.1

Words used in this chapter — dependency record, headline theorem, outside the dependencies, chain of the existence theorem. Their meanings are in the A-02 glossary.

Next chapter to read: 12-01 Lattice reflection positivity — two kinds of plane — on to Part 12, infinite volume and reconstruction.