Volume 2 Existence — reading the published construction

Part 13 Evidence that it is Yang–Mills

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The stress tensor lies on a single line

This chapter is established knowledge (a reading of the published construction). § 07 is this book's own investigation, and § 08 is a map of open questions.

What this chapter shows — That the published construction has a stress tensor T_{ab} with values in the field space F_4; that the candidates for such a tensor form a one-dimensional real line; and that the translation Ward identity picks out one point on it. The Ward identity says that "the flux of stress through a surface enclosing a point generates coordinate translations", in a form that does not touch coinciding points.

Prerequisite chapters — 8-03 (the field spaces F_d), 8-05 (correlations and phases). The classical action and conservation laws are in 2-01 and 3-03.

Said first — The stress tensor **does not appear in the published statement**. It is on the side of the construction's theorem and on the side of the committee's joint goal. How it is used to obtain Euclidean invariance is in 12-06. This chapter reads only the definitions and the form of the statements.

Labels: Source = we read the source of the published construction; Inferred = this book's inference; Assessment = from the committee's assessment. The other labels (Proved here, Cited, Reading, Lean and so on) mean the same as in the rest of the book (details in "Labels" in 6-00).

The order of this chapter

1. Statement — the conditions on the stress tensor and the one-dimensional line
2. The translation Ward identity: window functions and surfaces enclosing a point
3. The rotation Ward identity: flux weighted by the lever arm
4. Outline of the mechanism — why a line, and what picks out one point
5. What the committee's check confirmed and what it does not say
6. Which goal the statements belong to
7. What this book's investigation had seen

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01

Statement — the conditions on the stress tensor and the one-dimensional line

In classical Yang–Mills theory the stress tensor (energy–momentum tensor) is obtained by varying the action with respect to the metric (3-03). In quantum theory, pointwise products of fields have no meaning, so it cannot be defined that way. Assessment § 7.1 writes that the construction's stress tensor is **uniquely specified** by the following properties Assessment .

> The symmetric tensors J_{ab} with components in the field space F_4 that are unchanged by conjugation of fields, centred with respect to S^{mix} (one-point function 0), transform as rank-two tensors under coordinate permutations and sign flips, and have divergence 0 in F_5, form a one-dimensional real vector space. The stress tensor is a vector in this one-dimensional space, and its normalisation is fixed by the Ward identity.

The definitions on the construction's side are as follows Source (`YM/Euclid/Stress.lean` in `yang-mills-group/OpenAI-YM-20261028`).

- Type of a stress: $J : \{0,1,2,3\}^2 \to F_4$ (`Stress`, line 57).
- Conservation (`Conserved`, lines 59–62): symmetric $J_{ab} = J_{ba}$, real (invariant under conjugation), divergence zero $\sum_a \partial_a J_{ab} = 0$. Here $\partial_a : F_d \to F_{d+1}$ is the derivative map on fields, and the divergence is an element of F_5.
- Transformation as a tensor: a rank-two tensor under the hypercubic group (signed permutations) (`RawTensor`, line 68). The construction's stress is also a rank-two tensor under the action of the four-dimensional orthogonal group (`Tensorial`, lines 64–66).

The construction's stress tensor is the witness field `energy` (`YM/Spec/Closed.lean` line 31), and within the bundle of local laws `Locals` it has three properties, `conserved`, `spin` and `charges` (lines 41–43) Source .

One-dimensionality. The unconditional form is the field `ranks` of the structure `Pert.FirstExtension`, which enters the conclusion of the construction's theorem Source (`YM/Pert/FirstTarget.lean` lines 35–36, `Construction.lean` line 31): for the witness's derivative and rotation data, the real dimension of the kernel of the

divergence map is 1. The same fact also exists as the theorem `YM.Pert.one_line` (line 68) under the hypothesis `Supplier` (a bundle of ultraviolet data, Maxwell-field data, a sequence of scales and so on; `YM/Pert/Weak.lean` lines 22–32) Source . The kernel space (`KernelSpace`, `YM/Next/UV_STRESS/Tensor.lean` lines 75–82) is the space of the symmetry, reality and tensor conditions with "expectation 0 (centred)" added. Under the same hypothesis, `centered_line` (line 82) says that any J that is conserved, transforms as a tensor and satisfies the Ward laws becomes, once centred, a **non-zero real multiple** of the stress `I.J` in the field of `Supplier` Source . We have not read that `I.J` is the same as the witness's stress `energy`.

$$\Big\{J:\ \text{symmetric, real, centred, tensor, } \textstyle\sum_a \partial_a J_{ab} = 0\Big\} = \mathbb{R}\cdot T \quad (1)$$

02

The translation Ward identity

What picks out one point on the line (1) is the translation Ward identity. Take pairwise distinct points $x_0, x_1, \dots, x_n$, a central field $B \in F_d$ and spectator fields $q_1, \dots, q_n$, and choose a **window function** χ as follows Source (`Window`, `Stress.lean` lines 72–73): smooth with compact support, equal to 1 near x_0, and with none of the other points $x_1, \dots, x_n$ in its support. Then

$$-\sum_{a=0}^{3}\int_{\mathbb{R}^4} \partial_a\chi(y)\,\big\langle T_{ab}(y)\,B(x_0)\,q_1(x_1)\cdots q_n(x_n)\big\rangle\,d^4y = \frac{\partial}{\partial x_0^b}\big\langle B(x_0)\,q_1(x_1)\cdots q_n(x_n)\big\rangle \quad (2)$$

holds Assessment (equation (13) in Assessment § 7.1) Source (`StressLaws.translation`, lines 92–94; the right-hand side is first written as "the correlation of the differentiated field $\partial_b B \in F_{d+1}$", and that this equals the coordinate derivative is a separate law).

How to read it. $\partial_a\chi$ is 0 near x_0 ($\chi \equiv 1$) and also 0 near the other points ($\chi \equiv 0$), so the integrand on the left-hand side of (2) does not touch coinciding points. The "step" of χ is a thick shell enclosing x_0, and $-\int \partial_a\chi\,T_{ab}$ is the flux of stress through that shell. In classical terms, by Gauss's theorem

$$-\int \partial_a\chi\,T_{ab}\,d^4y = \int \chi\,\partial_a T_{ab}\,d^4y \quad (3)$$

and since $\partial_a T_{ab} = 0$ holds away from the insertion points, the right-hand side picks up only the "contact contribution" at x_0 Inferred . In quantum theory the right-hand side touches coinciding points, so it cannot be written. (2) is the form that fixes that contribution as the translation generator ∂_b, using only correlations at separated points. In the assessment's words, "the flux of stress around a point generates coordinate translations" Assessment .

Since (2) is linear, replacing T by cT multiplies the left-hand side by c. The right-hand side of (2) is not identically 0 (in some configuration the derivative of the correlation is non-zero), so on the line (1) only the single point $c = 1$ satisfies (2), and this is the normalisation Proved here .

03

The rotation Ward identity

An identity of the same form for rotations is also in the construction's bundle of laws Source (`StressLaws.rotation`, lines 95–97; the shell integral `shellSpin`, lines 84–86). For a four-dimensional antisymmetric matrix g (a rotation generator),

$$-\sum_{a,b}\int \partial_a \chi(y)\,(g\,(y-x_0))_b\,\langle T_{ab}(y)\,B(x_0)\cdots\rangle\,d^4y = \langle (A_g B)(x_0)\cdots\rangle \quad (4)$$

where A_g is the derivative of the rotation action on the field spaces (`Rotation.lie`, line 24). The flux weighted by the lever arm from x_0, as a flux of angular momentum, generates the rotation of the field. The main text of the assessment does not mention the rotation identity, but it is in the field `charges` of the construction's statement `Locals` Source .

The docstring of the definition says that this law "does not test contact terms at the point of B" (lines 88–89) Source .

04

Outline of the mechanism — why a line, and what picks out one point

We write down the intuition for why the candidates form a one-dimensional space Inferred . If one builds symmetric rank-two tensors of weight 4 from the curvature, the main ones are $\sum_c \mathrm{Tr}(F_{ac}F_{bc})$, $\delta_{ab}\sum_{c,e}\mathrm{Tr}(F_{ce}F_{ce})$ and $\delta_{ab}\mathbf{1}$. Centring fixes the $\mathbf{1}$ part, and the conservation condition constrains the remaining combination. For the classical Maxwell field, the only conserved one is the energy–momentum tensor (up to a constant multiple).

The Lean proof does not take the form of proving this intuition directly in the interacting theory. The proof of `one_line` calls `differential_rank`, whose arguments are the Maxwell-field data, the sequence of scales, and its limit and fit (`limit_`, `fit`) Source . It appears to count the dimension of the kernel through a comparison with the free Maxwell field along a sequence of short-distance scales (13-03 § 05) Inferred . We have not read the content.

To summarise the flow:

1. Define the field spaces F_4, F_5 and the derivative maps ∂_a via correlations (8-03).
2. Show that the space of J satisfying the symmetry, reality, tensor and conservation conditions is one-dimensional over the reals (`ranks`; the form with hypotheses is `one_line`).
3. Have the construction's stress tensor T as the point on that line satisfying the translation Ward identity (2) (`energy` and `charges`).
4. Use T to lift Euclidean invariance in the continuum limit (12-06). Take the trace and study the trace anomaly (13-02).

05

What the committee's check confirmed and what it does not say

The committee's annotated example `TranslationWard-20261028.lean` (in `submissions/openai-20261028/docs/examples/` of `yang-mills-group/YM` on the assessment side, revision 55ba9fc) proves four things in Lean about the translation Ward identity Assessment .

What it confirmed	Where
Admissible windows χ actually exist (the statement is not vacuous). The size of the window may depend on the configuration	`admissible_shell_exists`, line 40
The right-hand side of (2) equals the ordinary coordinate derivative, in every phase ξ, for central fields B of every degree and every list of spectators	`tensor_generates_translations`, line 64
The integrand of (2) is actually integrable (the integral does not rely on Lean's default value)	`shell_integrand_is_integrable`, line 85
The left-hand side of (2) does not depend on the choice of window	`shell_charge_is_window_independent`, line 102

The same example stresses that the same tensor is used for every central field, spectator and phase, without readjusting coefficients for each correlation (lines 12–14). Assessment § 11 also writes that the Ward identity translator covers all local fields and all components and checks the normalisation Assessment .

What it does not say Assessment (lines 111–122 of the same example):

- That all **improvement terms** of the stress tensor (additions that do not change the translation charge) vanish.
- A construction of contact terms extended to coinciding points.
- The dilatation Ward identity.
- A comparison with an independently chosen Minkowski sign convention.

Also, (2) is a normalisation relative to the Euclidean conventions for coordinates and derivatives; it is not an independent check of any further physical normalisation.

06

Which goal the statements belong to

Goal	What is there about the stress tensor
The published theorem `JointExistence`	**Nothing.** The published hierarchy has only the trace field `centeredTrace` (an element of F_4), and it does not even say that it is the trace of the stress (13-02 § 01) Source
The construction's theorem `constructed`	Its conclusion contains `Pert.FirstExtension F X` (`Construction.lean` line 31), whose field `ranks` (`YM/Pert/FirstTarget.lean` lines 35–36) states the dimension 1 in (1). The stress tensor and the Ward laws are in the fields of the witness F Source
The committee's joint goal	It includes the translation Ward check ([A6] of the assessment). It is also confirmed that the Ward identities, the scalar OS check and the trace law concern the same stress tensor and the same phase Assessment (Assessment § 9.3, § 9.4)
Headline theorems in the verification records	The record for the declaration `YM.Pert.one_line` is used, and the first headline found is `YM.Pert.Parity.parityEndpoint`. We have not checked the chain to the existence theorem Source (`verification/declaration-usage.tsv`)

"used" in the verification records means "reached from one of the seventeen headline theorems", not that the existence theorem uses it.

07

What this book's investigation had seen

This book's research and Lean have **neither** a stress tensor nor Ward identities. This book's objects are the Wilson measure on the lattice and coarse-step observables; it has no space of local fields (8-03), so there was nowhere to put a stress tensor.

On Euclidean invariance, this book went only as far as carrying the symmetry of the lattice's hypercubic group over to the pushforward measure (symmetric averaging of the block map), and placed invariance under continuous rotations among the statements to be shown (19-07). The idea of the published construction, lifting rotational invariance from "the Ward identities of the stress tensor and the vanishing of the flux at infinity", was not in this book (14-05).

08

What we have not read

- The definitions of the kernel space of `one_line` (`KernelSpace`) and of the divergence map `currentDiv`, and `differential_rank` in the proof (beyond lines 68–88 of `YM/Pert/Weak.lean`).
- The source of the "law of derivatives" that ties the right-hand side of Ward to the coordinate derivative (`F.phase.phase.calculus` in the committee's example).
- Where in the proof of Euclidean invariance the rotation Ward identity is used (within the scope of 12-06).
- From which observables on the lattice side the construction's stress tensor `energy` is built (the construction of the stress tensor itself).

What this chapter can say

Can say	Cannot say
Source The space of F_4-valued tensors satisfying the symmetry, reality, tensor and conservation conditions is one-dimensional over the reals (`ranks` in the conclusion of the construction's theorem; `one_line` under the `Supplier` hypothesis)	A stress tensor in the published statement (there is none)
Assessment The translation Ward identity (2) holds in every phase and for every field, does not depend on the window, and its integrand is integrable	That all improvement terms vanish; the dilatation Ward identity
Source The rotation Ward identity (4) is in the construction's bundle of laws	Contact terms (coinciding points)
Proved here By the linearity of (2), the normalisation on the line is fixed to one point	An independent comparison with the Minkowski sign convention

Sources and reproduction

Item	Kind	Source / tool
Conditions on the stress, windows, Ward laws	Source	`YM/Euclid/Stress.lean` lines 22–24 and 57–97 and `YM/Spec/Closed.lean` lines 27–43 of `yang-mills-group/OpenAI-YM-20261028` (Apache-2.0, revision 3456e5e)
One-dimensionality	Source	`YM/Pert/Weak.lean` lines 68 and 82, `YM/Pert/FirstTarget.lean` lines 26–39, `Construction.lean` lines 15–31, `verification/declaration-usage.tsv`

Item	Kind	Source / tool
Specification of the stress tensor, equation (13), the joint goal	Assessment	The assessment, `yang-mills-group/YM` `docs/assessment-1.pdf` (2026-11-10), § 7.1, § 9.3, § 9.4, § 11
The committee's Ward check	Assessment	`submissions/openai-20261028/docs/examples/TranslationWard-20261028.lean` on the assessment side, lines 12–14, 40, 64, 85, 102, 111–122

Terms used in this chapter — stress tensor, Ward identity, window function, flux, improvement term, contact term. Their meanings are in the A-02 glossary.

Next chapter: 13-02 The trace anomaly, and not being free — how the trace of the stress tensor behaves at short distances.

The trace anomaly, and not being free

This chapter is established knowledge (a reading of the published construction). § 07 is this book's own investigation, and § 08 is a map of open questions.

What this chapter shows — That in the chosen phase ξ^*, the two-point function of the centred trace field Θ behaves as $r^8 \log^2(1/r)\,\langle \Theta(-\frac{r}{2}e_0)\Theta(\frac{r}{2}e_0)\rangle \to 3\dim_{\mathbb{R}}\mathfrak{g}/\pi^4$; that the coefficient comes from the free two-point function of the curvature and the one-loop running; and that this logarithm excludes descriptions of the type "polynomials in free fields with finitely many masses".

Prerequisite chapters — 13-01 the stress tensor, 8-05 correlations and phases, 8-01 conventions (b_0 and $C_{\mathfrak{g}}$). Asymptotic freedom is in 4-06, dimensional transmutation in 4-07.

Said first — All the published statement says about the trace field Θ is that it is an element of F_4. Θ is one element inside an existential quantifier ($\exists T$), and what field it is is not written. That it is the trace of the stress tensor is written on the side of the construction's theorem and on the side of the committee's joint goal (§ 05). Also, what this law excludes is only one type of free-field description; it does not exclude all descriptions built from Gaussian fields.

Labels: Source = we read the source of the published construction; Inferred = this book's inference; Assessment = from the committee's assessment. The other labels (Proved here, Cited, Reading, Lean and so on) mean the same as in the rest of the book (details in "Labels" in 6-00).

The order of this chapter

1. Statement — the four fields of the trace test
2. Where the coefficient 3 dim g/π⁴ comes from: the free two-point function and the trace-anomaly formula
3. Outline of the proof of the short-distance law: approach through lattice points and squeeze by monotonicity
4. Not being free — what is excluded and what is not

5. The Θ of the published statement is one element of F_4: where being the trace of the stress is stated
6. Which goal the statements belong to
7. What this book's investigation had seen
8. What we have not read

01

Statement — the four fields of the trace test

The published hierarchy `Hierarchy` has one element `centeredTrace` of F_4 (below, Θ) Source (`ChallengeSpec.lean` line 48 in `yang-mills-group/OpenAI-YM-20261028`). In the chosen phase ξ^* (`selected`), put Θ at the two points $\mp\frac{r}{2}e_0$ on the time axis and let the two-point function be

$$G_\Theta(r) = S^{\xi^*}\left(\Theta(-\tfrac{r}{2}e_0)\,\Theta(\tfrac{r}{2}e_0)\right) \quad (1)$$

(`timePair`, `traceTwo`, lines 145–149). The published property `nonfree` consists of the four fields of the structure `TraceTest` Source (lines 168–175).

1. `centered`: in the reconstructed vacuum, $\langle\Omega, \Phi_\Theta(f)\Omega\rangle = 0$ for every Schwartz test function f.
2. `amplitudePositive`: $0 < 3\,\dim_{\mathbb{R}}\mathfrak{g}/\pi^4$. **An inequality between constants that does not involve the theory**, placed there so that it can be used in downstream proofs Assessment (Appendix A). It says nothing about the theory.
3. `shortDistance`:

$$\lim_{r\downarrow 0} r^8 \log^2(1/r)\, G_\Theta(r) = \frac{3\,\dim_{\mathbb{R}}\mathfrak{g}}{\pi^4} \quad (2)$$

4. `excludes`: it does not agree near 0 with any function W of the type in § 04.

(2) is the **full limit** as $r \downarrow 0$ (the filter $\mathcal{N}[>]0$), not a limit along a subsequence. Lengths are measured in the construction's fixed unit (the spacing at step 0, 8-02) Assessment (equation (16) in Theorem 7.1 of the assessment). For $SU(2)$, $\dim\mathfrak{g} = 3$ and the right-hand side is $9/\pi^4 \approx 0.0924$ Proved here .

The assessment also states, from Euclidean covariance, the form in all directions, $|x|^8 \log^2(1/|x|)\,K_\Theta(x) \to 3\,\dim\mathfrak{g}/\pi^4$ ($x \to 0$, $K_\Theta(x) = \mathrm{Re}\,S^{\xi^*}_{\Theta,\Theta}(x,0)$), and writes that the

committee's short-distance translator proved it Assessment (equation (17), § 11).

02

Where the coefficient 3 dim g/π⁴ comes from

Classical Yang–Mills theory is conformally invariant in four dimensions, and the trace of the stress tensor is 0. In quantum theory a trace remains because of renormalisation; this is called the trace anomaly. Assessment § 7.2 writes that "since the trace vanishes for smooth connections, this result may look strange, but Θ is in general non-zero as a result of renormalisation" Assessment .

The value of the coefficient is computed, as an informal comparison with perturbation theory, in § 5 (lines 195–236) of the committee's annotated example `LeadingTrace-20261028.lean` Assessment . We follow the argument.

(a) The free two-point function of the curvature. With the gauge field canonically normalised, set $Q = \sum_a \sum_{\mu,\nu} F^a_{\mu\nu} F^a_{\mu\nu}$ (summing over both orders of μ, ν). The free propagator of the field strength is

$$\langle F^a_{\mu\nu}(x) F^b_{\rho\sigma}(0)\rangle = \delta^{ab}\,\frac{I_{\mu\rho}I_{\nu\sigma} - I_{\mu\sigma}I_{\nu\rho}}{\pi^2|x|^4}, \qquad I_{\mu\nu} = \delta_{\mu\nu} - \frac{2x_\mu x_\nu}{|x|^2} \quad (3)$$

(equation (5.15) of Osborn–Petkou, cited by the example). By Wick's theorem the connected part of $\langle QQ\rangle$ is the sum of the two crossed contractions, and using the squared norm of the antisymmetric tensor, $\sum_{\mu\nu\rho\sigma}(I_{\mu\rho}I_{\nu\sigma} - I_{\mu\sigma}I_{\nu\rho})^2 = 2 \cdot 4 \cdot 3 = 24$, one gets

$$\langle Q(x)Q(0)\rangle_{\text{free},c} = 2 \cdot 24 \cdot \frac{\dim \mathfrak{g}}{\pi^4|x|^8} = \frac{48\,\dim \mathfrak{g}}{\pi^4|x|^8} \quad (4)$$

Proved here (since I is an orthogonal reflection matrix, $\sum_{\mu\rho} I^2_{\mu\rho} = 4$; the sum of squares is $16 + 16 - 2 \cdot 4 = 24$; we also checked this numerically).

(b) The trace-anomaly formula. With $\beta(g) = dg/d\log M$ (M the renormalisation scale), $\sum_a T_{aa} = \frac{\beta(g)}{2g}[Q]$ (equation (4.3) of Zoller–Chetyrkin translated to this $\beta(g)$; cited by the example). At one loop $\beta(g) = -b_0 g^3$, and the running coupling is $g^2(1/|x|) \sim 1/(2b_0\ell)$ ($\ell = \log(1/(\Lambda|x|))$), so

$$\frac{\beta(g)}{2g} = -\frac{b_0 g^2}{2} \sim -\frac{1}{4\ell} \quad (5)$$

Proved here .

(c) Combining. Multiplying (4) by the square of (5), $1/(16\ell^2)$, gives

$$\langle \Theta(x)\Theta(0)\rangle_c \sim \frac{48}{16}\cdot\frac{\dim \mathfrak{g}}{\pi^4|x|^8\ell^2} = \frac{3\dim\mathfrak{g}}{\pi^4|x|^8\ell^2} \quad (6)$$

Proved here . The finite correction from subtracting the one-point function does not change the leading singularity, and changing Λ shifts ℓ only by a constant.

To summarise how to read it:

- $|x|^{-8}$ is the naive power for the two-point function of a field of dimension 4.
- $\log^{-2}$ is the trace of the fact that the trace has the form $g^2\cdot F^2$, and g^2 becomes small like $1/\log$ by asymptotic freedom.
- **The numerical value of b_0 drops out of the coefficient.** This is because the b_0 in (5) cancels the b_0 in the running. So this law alone cannot confirm the value $b_0 = 11C_{\mathfrak{g}}/(48\pi^2)$. That value is fixed by other results (the effective coupling in 13-03, the running in 10-06) Assessment (Assessment § 7.2, lines 230–236 of the example).

On the Lean side, the coefficient is first written in the form $12\,(d_2^2/b_0^2)\dim\mathfrak{g}/\pi^4$ and simplified to $3\dim\mathfrak{g}/\pi^4$ using the identity $d_2 = -b_0/2$ (`Trace.Over.force_b`) Assessment (lines 170–177 of the example, `coefficient_is_fixed` line 261). $12\cdot\frac{1}{4} = 3$ Proved here . The example notes that this algebraic simplification is only the last step of the substantive short-distance analysis.

03

Outline of the proof of the short-distance law

Appendix A of the assessment (the entry for `TraceTest.shortDistance`) describes the skeleton of the proof as follows Assessment . Put $c(r) = r^8\log^2(1/r)$.

1. **Approach through lattice points.** For some sequence $r_n\downarrow 0$, at distances $r_n\delta_n(s)$ ($\delta_n(s)\to s$) that approximate the physical distance $r_n s$ (s a constant near 1) by nearby lattice points, show $c(r_n)\,G_\Theta(r_n\delta_n(s))\to A/s^8$ ($A = 3\dim\mathfrak{g}/\pi^4$) from the ultraviolet comparison, the logarithmic normalisation and the computation of the free two-point function.
2. **Monotonicity.** By the positive spectral representation (which comes from reflection positivity), G_Θ on the time axis is real and non-increasing.
3. **Squeeze.** Take $s_- < 1 < s_+$; eventually $G_\Theta(r_n\delta_n(s_+)) \le G_\Theta(r_n) \le G_\Theta(r_n\delta_n(s_-))$. Multiply by $c(r_n)$ and let $n\to\infty$, then $s_\pm\to 1$.

The committee's example notes that the subsequence used inside the proof does not weaken the conclusion to convergence along a subsequence Assessment (`LeadingTrace-20261028.lean` lines 186–193; `YM.Trace.Verse.outside` lifts the analysis along the controlled subsequence to the full limit over positive distances). The declarations the example lists as a reading path are `YM.Trace.wick_norm` (the Wick computation of the free two-point coefficient), `YM.Trace.Over.Scribe` (the scale comparison between the free field and the logarithmic clock), `YM.Trace.Verse.tagline` and `YM.Trace.Verse.border`. This book has not read their content.

04

Not being free — what is excluded and what is not

The excluded type. Take a mass $m \geq 0$, a time order $q \geq 0$ and a continuous homogeneous (of degree $j \geq 0$) real function h, and consider the kernel

$$K_{m,q,h}(r) = \int_{\mathbb{R}^3} \frac{E_m(p)^q\, h(p)}{2E_m(p)}\, e^{-rE_m(p)}\, d^3p, \qquad E_m(p) = \sqrt{m^2 + |p|^2} \tag{7}$$

Source (`Mode`, `Mode.kernel`, `ChallengeSpec.lean` lines 152–161) Assessment (equation (18) in Assessment § 7.3). This is the time-direction representation of the two-point function (and its derivatives) of a free field of mass m. For finite complex linear combinations of finite products of such kernels,

$$W(r) = \sum_{a=1}^{N} c_a \prod_{b=1}^{n_a} K_{m_{ab},q_{ab},h_{ab}}(r) \tag{8}$$

(`wickProfile`, lines 165–166), `excludes` says "for every W and $\delta > 0$, there is $r \in (0, \delta)$ with $G_\Theta(r) \neq W(r)$" Source (line 175; equivalent to the assessment's wording Assessment). (8) includes the two-point functions of polynomial observables (including derivatives) of free fields with finitely many masses.

Why they are excluded Assessment (the entry for `excludes` in Appendix A). The construction shows that the real part of any W can be expanded near 0 as

$$\operatorname{Re} W(r) = \sum_{\nu} a_\nu\, r^{\alpha_\nu} (\log r)^{k_\nu} + O(1), \qquad \alpha_\nu \in \mathbb{Z},\ k_\nu \in \mathbb{Z}_{\geq 0} \tag{9}$$

and shows that the only possible finite limit of $r^8 \log^2(1/r) \operatorname{Re} W(r)$ is 0. If $W = G_\Theta$, this contradicts the positive limit in (2). The intuition is that no term of (9) behaves like $r^{-8}(\log r)^{-2}$ Inferred : the powers of the logarithm are non-negative, so each term multiplied by $r^8 \log^2$ either goes to 0 or diverges, and if the limit of the sum is finite it is 0. Terms of the form $\log^{-2}$ do not appear in (9).

What is not excluded Assessment (Assessment § 7.3, `LeadingTrace-20261028.lean` lines 337–341):

- All descriptions built from Gaussian fields (the assessment states explicitly that "it does not exclude all of them").
- All generalised free fields.
- Determination of the intrinsic mass scale Λ. Agreement with perturbation theory to all orders. The structure of the vacuum.

The positive leading coefficient excludes that the trace correlations are identically 0. The assessment derives in particular that the two-point function is non-zero for sufficiently small r, and that some smeared $\Phi\Theta(f)$ is non-zero Assessment .

The stronger form on the side of the construction's theorem, `Headline.TraceExclusions` (`YM/Nonperturbative/HeadlineSpec.lean` lines 84–120), additionally has the fields `identified` (§ 05), `wick`, `maxwell` and `invariant` Source (as far as field names and types). We have not read the meaning of the fields from `wick` on.

05

The Θ of the published statement is one element of F_4

This is the point in this chapter that needs the most care.

Reading only the published statement, `centeredTrace` is one of the fields of the hierarchy T given by the existential quantifier $\exists T$ of the published theorem, and all the statement says about it is that its type is F_4 (`Field 4`) Source (`ChallengeSpec.lean` lines 48 and 186–188). What the published property `nonfree` says is that "there is a field Θ of degree 4, centred in the chosen phase, whose two-point function satisfies (2) and `excludes`". That it is the trace of the stress tensor is not written in the published statement.

Being the trace of the stress is stated in two places.

- **The construction's side.** In the definition that builds the published hierarchy, `centeredTrace := TraceExtension.center F` (`HierarchyBridge.lean` line 42) Source . The field `identified` (`HeadlineSpec.lean` lines 87–91) of the conclusion `Headline.TraceExclusions` of the construction's theorem says that the operator of the trace field equals "the trace field of the stress tensor `F.sector.energy` minus its one-point function in the chosen phase times the identity" Source . This is equation (14) of the assessment, $\Theta = \sum_\mu T_{\mu\mu} - S^{\xi^*}\left(\sum_\mu T_{\mu\mu}\right)\mathbf{1}$ Assessment .

- **The committee's joint goal.** The short-distance check ([A4] of the assessment) confirms that it uses the trace of the same stress tensor whose normalisation was checked by the Ward identity, in the same chosen phase Assessment (Assessment § 9.4, § 11; in the example `LeadingTrace-20261028.lean`, `centering_uses_the_phase_mean` line 61 and `source_and_OS_fields_agree` line 97).

Assessment § 2 writes that "the construction defines the trace of the stress tensor, `centeredTrace`, and proves its properties in `nonfree`". That is a reading that includes the construction's side; this book keeps the distinction that **"the trace of the stress" does not follow from the wording of the published statement alone**. In short, what the published theorem says is that "(2) holds for some field of degree 4", and it is the construction's theorem and the committee's joint goal that identify that field with the trace of the stress tensor.

Conditions also enter the choice of the chosen phase ξ^*. Appendix A of the assessment writes that the set of phases whose ground-state space is one-dimensional and that satisfy a certain limiting property of the trace field is shown to have measure 1 for the measure on phases (the probability measure (4) in 8-05), and that ξ^* is chosen from it Assessment (the entry for `Hierarchy.selected`).

06

Which goal the statements belong to

Goal	What is there
The published theorem `JointExistence`	The four fields of `TraceTest`. About Θ it says only that it is an element of F_4 Source
The construction's theorem `constructed`	`Headline.TraceExclusions` (including `identified`, `shortDistance`, `positive`, `wick`, `maxwell`, `invariant`) Source (`Construction.lean` line 28)
The committee's joint goal	The short-distance check [A4] (the all-directions form (17)), the independent check of the OS axioms for the hierarchy of the trace field [A1], the non-zero lattice limit [A5] ((5) in 8-05) Assessment

The committee's independent check of the OS axioms is carried out for **the hierarchy of Θ alone** (Assessment § 10, § 10.1), not for all fields Assessment .

07

What this book's investigation had seen

This book's research and Lean have **none** of the trace anomaly, the short-distance law or the exclusion of free fields.

- This book's viewpoint on non-triviality was on the infrared side (the sign and floor of the vortex ratio θ_M, pairs of Polyakov loops, non-degeneracy of face averages) (Part 21). The non-triviality of the published construction is on the ultraviolet side (the logarithm of asymptotic freedom). The two lie on complementary sides Inferred .
- The θ in this book's vortex ratio θ_M is a symbol unrelated to the trace field Θ.
- The idea of "stating non-freeness through the logarithm of the trace anomaly" was not in this book (14-05).

08

What we have not read

- The main body of the proof of (2) (`wick_norm`, `Scribe`, `tagline`, `border`, `outside`).
- The proof of the expansion (9) (`YM.TraceExtension.Wick.real_profile` and others).
- The meaning of the fields `wick`, `maxwell` and `invariant` of `TraceExclusions` (`HeadlineSpec.lean` lines 96–120).
- The proof that the set of chosen phases has measure 1 (`YM.OS.Cluster.lifts` and others).
- The propagator (3) and the trace-anomaly formula in (b) are formulas from the literature cited by the committee's example; this book has not checked the original sources (bibliographic entries only in the table of sources).

What this chapter can say

Can say	Cannot say
Source The published `nonfree` states, for an element Θ of F_4 centred in the chosen phase, the full limit (2) and the exclusion of the free-field type	"Θ is the trace of the stress" from the published statement alone
Source Being the trace of the stress is in the construction's theorem (`identified`); Assessment that it is the same stress and the same phase is in the committee's joint goal	That the independent OS check exists for all fields (only for the hierarchy of Θ)
Proved here The count of the coefficient $48/16 = 3$ (free two-point function $\times$ the square of the one-loop coefficient of the trace anomaly). b_0 drops out	Confirmation of the value of b_0 by this law

Can say	Cannot say
`Assessment` The type of two-point functions of polynomials in free fields with finitely many masses is excluded	Exclusion of all descriptions built from Gaussian fields or of all generalised free fields; the intrinsic scale; agreement with perturbation theory to all orders

Sources and reproduction

Item	Kind	Source / tool
Trace test, kernels, `wickProfile`	Source	`ChallengeSpec.lean` lines 48 and 145–175 of `yang-mills-group/OpenAI-YM-20261028` (Apache-2.0, revision 3456e5e)
Identification with the trace of the stress	Source	`HierarchyBridge.lean` line 42, `YM/Nonperturbative/HeadlineSpec.lean` lines 84–120, `Construction.lean` line 28
Theorem 7.1, equations (14)–(18), exclusion of free fields, outline of the proof, the chosen phase	Assessment	The assessment, `yang-mills-group/YM` `docs/assessment-1.pdf` (2026-11-10), § 2, § 7.2, § 7.3, § 9.4, § 10, § 11, Appendix A
Computation of the coefficient and the reading path	Assessment	`submissions/openai-20261028/docs/examples/LeadingTrace-20261028.lean` in `yang-mills-group/YM` on the assessment side, revision 55ba9fc, lines 61, 97, 170–236, 261, 337–341
Free propagator of the field strength (equation (5.15))	Cited	H. Osborn, A. C. Petkou, Ann. Phys. 231 (1994) 311, arXiv:hep-th/9307010 — bibliographic entry only (not checked; cited by the committee's example)
Trace-anomaly formula (equation (4.3))	Cited	M. F. Zoller, K. G. Chetyrkin, JHEP 12 (2012) 119, arXiv:1209.1516 — bibliographic entry only (not checked; cited by the committee's example)
Computation improved by the position-space renormalisation group	Cited	M. Bochicchio, S. P. Muscinelli, JHEP 08 (2013) 064, arXiv:1304.6409 — bibliographic entry only (not checked; cited by the committee's example)
One-loop coefficient	Cited	D. J. Gross, F. Wilczek, Phys. Rev. Lett. 30 (1973) 1343; H. D. Politzer, Phys. Rev. Lett. 30 (1973) 1346
The counts 24 and 48/16	Proved here	Numerical record of this book's investigation (`numcheck.py`; counted the sum of squares in (4) with random reflection matrices)

Terms used in this chapter — trace anomaly, centred trace, short-distance law, chosen phase, exclusion of free fields, Wick product. Their meanings are in the A-02 glossary.

Next chapter: 13-03 Free Maxwell fields at short distance — how the theory approaches free Maxwell fields at short distances.

Free Maxwell fields at short distance

This chapter is established knowledge (a reading of the published construction). § 07 is this book's own investigation, and § 08 is a map of open questions.

What this chapter shows — That asymptotic freedom appears in two forms. One is that the coupling $g_W(r)$ read from the effective action behaves as $\log(1/r)\, g_W(r)^2 \to 1/(2b_0)$ (Theorem 7.2 of the assessment). The other is that, rescaling the correlations along a certain common sequence $r_k \downarrow 0$, they approach the correlations of $\dim \mathfrak{g}$ independent free Maxwell fields, with derivatives of every order.

Prerequisite chapters — 8-02 (steps and spacings), 13-01 (the stress tensor). The definition of "the coupling at step m" is in 10-01, and the running of the bare coupling in 10-06. The introduction to asymptotic freedom is 4-06.

Said first — The assessment does not give a Lean name for Theorem 7.2 (the effective coupling). Statements with similar content exist in three places (§ 03): `Running.ratio`, outside the dependencies; `wilson_link`, inside the dependencies and used as an intermediate step in the trace analysis (its wording is close to the assessment's statement); and the headline theorem `YM.Pert.Asymptotics.comparison` (the running of the coupling of the cut-off steps, a different quantity from g_W). A statement in the form of g_W does not survive into the conclusions of the published theorem or of the construction's theorem. It is not in the list of checks of the committee's joint goal either. The comparison with Maxwell fields is in the conclusion of the construction's theorem. Neither claims agreement with perturbation theory to all orders.

Labels: Source = we read the source of the published construction; Inferred = this book's inference; Assessment = from the committee's assessment. The other labels (Proved here, Cited, Reading, Lean and so on) mean the same as in the rest of the book (details in "Labels" in 6-00).

The order of this chapter

1. Statement — the three results of Assessment § 7.4

2. Definition of the effective coupling, and how the running is derived
3. Where the effective coupling is in Lean, and the goals: outside and inside the dependencies
4. The second coupling — measured by continuum correlations
5. Comparison with free Maxwell fields: along a common sequence, with derivatives of every order
6. How the three short-distance results differ
7. What this book's investigation had seen
8. What we have not read

01

Statement — the three results of Assessment §7.4

Assessment § 7.4, "Asymptotic freedom and agreement with perturbation theory", lists three results Assessment .

1. **Effective coupling (Theorem 7.2).** Integrating out fluctuations finer than a length r changes the coefficient of the curvature-squared term in the effective action. Its inverse square root, in a fixed normalisation, is the running coupling. The submission starts from the family of Wilson theories with the bare coupling of Theorem 4.1, blocks down to the first lattice spacing below r, sets the initial cutoff to 0, and after taking the limit along a subsequence of volumes defines $g_W(r)$ as the inverse square root of the normalised coefficient of the action. For small r, and for the limit along any chosen subsequence of volumes, $g_W(r) > 0$ and

$$\log(1/r)\, g_W(r)^2 \longrightarrow \frac{1}{2b_0} \qquad (r \downarrow 0), \qquad b_0 = \frac{11 C_{\mathfrak{g}}}{48\pi^2} = \frac{s}{2} \quad (1)$$

   (In this chapter b_0 is the one-loop coefficient, and $2b_0 = s$ is the one-loop slope. It is not the block side b_i at $i = 0$.) It has not been shown that $g_W(r)$ does not depend on the choice of subsequence of volumes.
2. **The second coupling.** A coupling is defined from continuum correlations at separated points, and its ratio to g_W tends to 1 as $r \downarrow 0$. The running law is visible not only in the effective action but also in continuum observables.
3. **Comparison with free Maxwell fields.** Along one common sequence $r_k \downarrow 0$, the averaged n-point correlations of the stress tensor, multiplied by r_k^{4n} and evaluated at $r_k x_1, \dots, r_k x_n$, approach the correlations of $\dim \mathfrak{g}$ independent free Maxwell fields, with derivatives up to any fixed order, uniformly on compact sets avoiding

coinciding points. Here a Maxwell field is the Gaussian 2-form curvature obtained as the exterior derivative of a Gaussian free $\mathfrak{g}$-valued 1-form.

The assessment distinguishes that the comparison with Maxwell fields is for the mixed theory along a particular sequence, that Theorems 7.1 (the trace law, 13-02) and 7.2 hold for any way of letting $r \to 0$, and that 7.1 concerns the chosen phase; and writes that together they form "the main short-distance evidence" Assessment .

02

Definition of the effective coupling, and how the running is derived

Here we read the definition in the form that is inside the dependencies (`Pilot.wilson_link` in § 03) Source (`YM/TraceExtension/Atlas/Sink.lean`, `YM/TraceExtension/Terrace/Dusk.lean` in `yang-mills-group/OpenAI-YM-20261028`).

The limiting coupling at step i. $\beta(N,p,i)$ is the coupling at step i of the trajectory with cutoff N and volume p (the coefficient of F^2 in the representation of the effective action; 10-01). Take one ultrafilter $\mathcal{L}$ on (N,p) (matched to the way the limit along a subsequence of volumes is taken), and take its limit

$$Z_\beta(i) = \lim_{\mathcal{L}} \beta(N,p,i) \quad (2)$$

at each step (`Arch`, `steep`, `Dusk.lean` lines 65–72). At each step $Z_\beta(i) \in [\hat\beta_i/2,\, 2\hat\beta_i]$ (`holds`, line 79), where $\hat\beta_i = \bar\beta + 2b_0 \log(1/a_i)$ is the one-loop running (8-02), and moreover $Z_\beta(i)/\hat\beta_i \to 1$ (`gamut`, line 89).

The step just below r. Let $i(r)$ be the first step with $a_i < r$ (`glen`, `Sink.lean` line 14). The effective coupling is

$$g_W(r) = Z_\beta\big(i(r)\big)^{-1/2} \quad (3)$$

(`wilson`, line 44).

Running. Multiply two limits together Source (statements) Proved here (combination).

- `pace_log` (line 21): $\log(1/r)/\hat\beta_{i(r)} \to 1/(2b_0)$. The spacing satisfies $a_{i(r)} < r \le a_{i(r)-1}$, and the proof uses the field `bloom` of the additional calendar condition `PostT` ($b_i \le \log \hat\beta_i$, `YM/TraceExtension/Terrace/Base.lean` lines 29–34) and the monotonicity of $\hat\beta$ to bound the difference between $\log(1/r)$ and $\log(1/a_{i(r)})$ at the order of $\log\hat\beta$ Source (`H.bloom` and `monoBeta` in `Sink.lean` lines 21–41).
- `gamut`: $Z_\beta(i)/\hat\beta_i \to 1$.

Dividing, $\log(1/r)\, g_W(r)^2 = \log(1/r)/Z_\beta(i(r)) \to 1/(2b_0)$ (`wilson_link`, line 50; the conclusion type `Coupling`, lines 45–47, has two parts: "eventually positive" and "$\log(1/r)\, u(r)^2 \to 1/(\text{slope})$"). Moreover $g_W(r) \to 0$ (`Coupling.conforms`, line 55). The hypotheses of `wilson_link` are that G is simple, the calendar condition `PostT`, the ultrafilter limit `steep`, and that the ultrafilter follows the order of extraction ($\mathcal{L} \leq X.\text{fuse}$) Source . `PostT` is a condition imposed on the side of the trace analysis.

Two cautions on how to read this.

- $Z_\beta(i)$ depends on the ultrafilter $\mathcal{L}$. This is what the assessment means by "it has not been shown not to depend on the choice of subsequence".
- $\beta(N, p, i)$ is a **coefficient in a representation** of the effective action, not an invariant that can be read from the measure alone (10-01). So g_W too is a quantity defined through the choice of representation in the construction.

03

Where the effective coupling is in Lean, and the goals

The assessment does not give a Lean name for Theorem 7.2. Statements with similar content exist in three places Source . The one whose wording is close to the assessment's statement ($r \downarrow 0$, the first spacing below r, eventually positive) is `wilson_link` Inferred .

Declaration	Content	Verification record (`verification/declaration-us`
`YM.PertExtension.Running.ratio` (`YM/PertExtension/Running.lean` line 145)	The ratio form $g(\mu) \cdot (2b_0 \log \mu)^{1/2} \to 1$ ($\mu \to \infty$). g is `gLedger` (line 130), the inverse square root of the value of the step limit B at the first step with $\log \mu \leq \log(1/a_m)$ ($a_m \leq 1/\mu$). Positivity holds for every μ (`ledger_positive`, line 139)	**outside** (outside the dependencies). The coupli (`isLimit` 59, `discrete` 66, `choose` 86, `band` 1 `gLedger` 130, `ledger_positive` 139, `ratio`, `va` 165 and so on) are all outside. What is used in t module is the extraction `extraction` (line 40) a `clock` (line 20), and the private helpers `cut` (l and `compact_diag` (line 27)

Declaration	Content	Verification record (`verification/declaration-us`
`YM.Trace.Pilot.wilson_link` (`YM/TraceExtension/Atlas/Sink.lean` line 50)	$\log(1/r)\, gW(r)^2 \to 1/(2b_0)$ $(r \downarrow 0)$, eventually positive. gW is (3). `PostT` ($b_i \le \log \hat{\beta}_i$ and so on) among the hypotheses	used. The first headline found is `continuum_with_position_tests`, and the direct `crest_ready`. An intermediate step in the trace
`YM.Pert.Asymptotics.comparison` (`YM/Pert/Asymptotics/RawAxis.lean` line 36)	For $N_i \ge i$ (eventually) and any sequence of volumes k_i, the coupling of the cut-off step i, $\beta(N_i, k_i, i)^{-1/2}$ (`regulatorRun`, line 35), multiplied by $\sqrt{\log(1/a_i)}$, tends to $(2b_0)^{-1/2}$. This is the running on the scale of steps before any limit is taken, a different quantity from $gW(r)$	**headline** (a headline theorem itself)

The docstring at the top of `Running.lean` says that the coupling in this module is "an auxiliary observation read from the represented action", and that the main equivalence "does not identify a position-space coupling" and "does not estimate the diagrammatic remainder" (lines 6–10).

The conclusion of `wilson_link` appears in the field `charts` of the intermediate structure `Crest` of the short-distance trace analysis (`YM/TraceExtension/Belfry.lean` lines 20–22), and in the field `wilson` of the comparison structure `FieldComparison` (`YM/TraceExtension/Endpoint.lean` line 50) Source . One can read this as the proof of the trace law (13-02) using the running of the coupling as an intermediate step Inferred . However, when passing from these structures to the published trace test (`Integration.ManuscriptTraceTests.of_result`), the comparison fields are dropped Source (`YM/Integration/ManuscriptTraceTests.lean` lines 62–89).

Summary with respect to the goals.

- The published theorem `JointExistence`: the effective coupling is not in it. What is there is only the running of the bare coupling, `WilsonRegularity.running` ((1) in 8-02) Source .

- The construction's theorem `constructed`: there is no statement in the form of g_W among the fields of its conclusion Source (`Construction.lean` lines 15–31). The conclusion's `Independent.Core` has the trace tests `ManuscriptTraceTests.Tests` in its field `trace` (`YM/Independent/Core.lean` lines 18–22), and `Trace.Target.Result`, which contains `FieldComparison` (`YM/TraceExtension/Endpoint.lean` lines 99–120), is dropped by `of_result`. The conclusion of `continuum_with_position_tests` has the same `Core` (`YM/Nonperturbative/Continuum.lean` lines 28–41) Source .
- The committee's joint goal: there is no coupling check in the list of checks in Assessment § 9.3 Assessment . The committee's example writes that the trace check and its example do not add an independent check of Theorem 7.2 Assessment (`LeadingTrace-20261028.lean` lines 233–236).
- Headline theorems: the running of the step coupling, `YM.Pert.Asymptotics.comparison`, is a headline theorem itself. In the form of g_W, `Running.ratio` is outside the dependencies and `wilson_link` is an intermediate step inside them Source .

04

The second coupling — measured by continuum correlations

According to the assessment, the second coupling is defined from continuum correlations at separated points Assessment : the derivative of the scalar two-point function and the integrated tensor–scalar–scalar correlation are normalised to their free values, and the coupling is measured by a derivative of the tensor–scalar correlation. The submission shows that the ratio of this coupling to g_W tends to 1 as $r \downarrow 0$.

On the Lean side this coupling is `Sundial.glo` (`YM/TraceExtension/Sundial/Noon.lean` line 28, used), and that the ratio tends to 1 is the field `calibrationX` of `FieldComparison` (`Endpoint.lean` line 49) Source . `FieldComparison` also has among its fields that the one-loop part of the coefficient is $-b_0/2$ (`coefficient_O`, `traced_O`: of the form $d(r) - d_2\, g^2 = O(g^4)$). We have not read the content.

05

Comparison with free Maxwell fields

The conclusion of the construction's theorem contains `Pert.FirstExtension F X` Source (`Construction.lean` line 31). X is a sequence of scales on the correlation table S (`Climb.Scaling`), with the common sequence $r_k = X.r\, k \downarrow 0$. The fields of the structure `FirstExtension` (`YM/Pert/FirstTarget.lean` lines 26–39) are as follows.

- `jets` (lines 29–34): for every number of points n, derivative order j, list of labels of the free table, marks of which positions are stresses together with their indices, compact set K avoiding coinciding points, and $\varepsilon > 0$, for all sufficiently large k and at every point of K,

$$\left\| D^j \left[z \mapsto \widetilde{S}_k(r_k z) \right](x) - D^j S_{\mathrm{free}}(x) \right\| \leq \varepsilon \quad (4)$$

  Here $\widetilde{S}_k$ is the rescaled correlation of the construction (`scaledPhysical`), and S_{free} is the correlation of the free table, with the stress of the free Maxwell field `freeStress` $= -$ `Free.original` (line 16) placed at the stress positions.
- `ranks` (lines 35–36): the line of candidates for the stress is one-dimensional (13-01).
- `derivatives`, `rotations` (lines 37–39): the derivatives and the rotation action on fields approach those of the free field along the sequence of scales.

(4) is the form, with derivatives of any order, of "at short distances the construction's theory looks like $\dim \mathfrak{g}$ independent free Maxwell fields". The docstring says that this is a first milestone concerning only the correlations of the mixed theory (the hierarchy), and that it "does not yet include estimates on the amplification of fixed labels or on the matched coupling" (lines 4–5, 21–25) Source . Fields at non-stress positions are measured in the frame `X.U k` for each element of the sequence of scales. The assessment's "including comparison with fields other than the stress tensor" can be read as this part Inferred .

We infer that the scale factor r_k^{4n} (in the assessment) enters Lean inside `scaledPhysical` (`YM/Pert/Lifted.lean` line 81), but we have not checked.

The verification record for the declaration `FirstExtension.jets` is used (first headline `parityEndpoint`), and the projection `ranks` is outside, but both are fields of the structure in the conclusion of the construction's theorem, so as statements they are in the construction's theorem Source .

06

How the three short-distance results differ

	Trace law (13-02)	Effective coupling (1)	Comparison with Maxwell fields (4)
How $r \to 0$ is taken	Full limit	Full limit	One common sequence r_k
State	Chosen phase ξ^*	Limit along a subsequence of volumes (ultrafilter)	Mixed theory

	Trace law (13-02)	Effective coupling (1)	Comparison with Maxwell fields (4)
What is measured	Two-point function of the centred trace	Coefficient in the representation of the effective action	Correlations (stress and other fields), derivatives of every order
Value of b_0	Drops out of the coefficient	Appears in $1/(2b_0)$	Does not appear (comparison with free fields)
Goal	Published (with Θ only as one element of F_4), construction, committee	The gW form is in no conclusion (the headline theorem is the running of the step coupling, `comparison`)	Conclusion of the construction's theorem

None of the three **claims agreement with perturbation theory to all orders** Assessment (Assessment § 7.5: "agreement with perturbation theory to all orders is unproved"). The two-loop coefficient and the $\log\log$ correction are not treated either (10-06).

07

What this book's investigation had seen

This book has **neither** a theorem on the effective coupling nor a comparison with free Maxwell fields.

- This book measured numerically how the coupling moves under one block step, and recorded that its direction agrees with asymptotic freedom and its size is 2.9–4.4 times the one-loop value in units of β, $\frac{11}{3\pi^2}\ln 2 \approx 0.2575$ (15-01, label Reading). The published construction's slope $11C_{\mathfrak{g}}/(24\pi^2)$ multiplied by $\ln 2$ is this one-loop value (8-01 § 04).
- This book's "matching" took the form of matching observables definable from the measure alone (expectations on coarse faces, the vortex ratio) to fixed values. The published construction's coupling is a coefficient in a representation, and (3) reads it as an effective coupling. The comparison of which of the two is meaningful in the continuum limit is in 14-03.

08

What we have not read

- The whole chain from `wilson_link` to the existence theorem (beyond `crest_ready`). The meaning of the other fields of `PostT` (`cropK`, `share`, `purse`, `jig`).
- The definition of the second coupling `glo` (`lampX`) and the proof of `calibrationX`.

- `scaledPhysical` and the scale factor, how the free table `freeTable` is built, and the meaning of the frame `X.U k`.
- Whether the limit B in `Running.lean` (the ordered extraction `f.fuse` of `isLimit`) includes the limit in volume.

What this chapter can say

Can say	Cannot say
Source Defining $g_W(r)$ by (2)(3), $\log(1/r)\, g_W(r)^2 \to 1/(2b_0)$ (`wilson_link`)	That g_W does not depend on the subsequence of volumes, or is determined by the measure alone
Source `Running.ratio` is outside the dependencies, and `wilson_link` is an intermediate step inside them. A statement in the form of g_W is not in the published or construction conclusions (the headline `comparison` is the running of the coupling of the cut-off steps, a different quantity)	The whole chain from `wilson_link` to the existence theorem
Source The conclusion of the construction's theorem contains the comparison with free Maxwell fields along a common sequence (derivatives of every order, uniformly on compact sets)	A comparison for every $r \to 0$; a comparison in the chosen phase
Assessment The ratio of the second coupling to g_W tends to 1	Agreement with perturbation theory to all orders; the two-loop coefficient

Sources and reproduction

Item	Kind	Source / tool
Effective coupling (inside the dependencies)	Source	`YM/TraceExtension/Atlas/Sink.lean` lines 14, 21, 44–55, `YM/TraceExtension/Terrace/Dusk.lean` lines 65–89, `YM/TraceExtension/Belfry.lean` lines 16–22 of `yang-mills-group/OpenAI-YM-20261028` (Apache-2.0, revision 3456e5e)
Effective coupling (outside the dependencies)	Source	`YM/PertExtension/Running.lean` lines 6–10, 40, 59, 66, 130, 145, 165; `verification/declaration-usage.tsv`
Second coupling, comparison structure	Source	`YM/TraceExtension/Sundial/Noon.lean` line 28, `YM/TraceExtension/Endpoint.lean` lines 36–64, `YM/Integration/ManuscriptTraceTests.lean` lines 62–89
Comparison with Maxwell fields	Source	`YM/Pert/FirstTarget.lean` lines 4–39, `Construction.lean` lines 15–31

Item	Kind	Source / tool
Theorem 7.2, the second coupling, the Maxwell comparison, agreement with perturbation theory	Assessment	The assessment, `yang-mills-group/YM` `docs/assessment-1.pdf` (2026-11-10), § 7.4, § 7.5; on the assessment side, `submissions/openai-20261028/docs/examples/LeadingTrace-20261028.lean` in `yang-mills-group/YM`, revision 55ba9fc, lines 230–236

Terms used in this chapter — effective coupling, running coupling, ultrafilter limit, free Maxwell field, sequence of scales, derivatives of every order. Their meanings are in the A-02 glossary.

Next chapter: 13-04 The operator product expansion and contact terms — the expansion of correlations as points approach each other, and the extension to coinciding points.

The operator product expansion and contact terms

This chapter is established knowledge (a reading of the published construction). § 07 is this book's own investigation, and § 08 is a map of open questions.

What this chapter shows — That an expansion approximating correlations of products of fields gathered near a point o by linear combinations of correlations of single fields placed at o — the operator product expansion — is in the published statement, in the form of power bounds on the coefficients and on the remainder. The remainder bound is uniform over the mixed theory and all phases, and the coefficients do not depend on the spectators, the state or the truncation degree. For the distribution extended to coinciding points, only that it "exists" is stated.

Prerequisite chapters — 8-03 (field spaces and degrees), 8-05 (correlations and phases), 13-02 (the short-distance logarithm).

Said first — D is a degree of fields, not a loop order. Convergence as $D \to \infty$, agreement of the coefficients with perturbative values, and independence of the coefficients from the choice of subsequence of volumes are not stated. Nor does equality of phases follow from the common coefficients.

Labels: Source = we read the source of the published construction; Inferred = this book's inference; Assessment = from the committee's assessment. The other labels (Proved here, Cited, Reading, Lean and so on) mean the same as in the rest of the book (details in "Labels" in 6-00).

The order of this chapter

1. Statement — conditions on the configuration and the two bounds
2. How to read the powers: dimension counting and the loss ε
3. How the coefficients and channels are built
4. Uniformity over states, and what the common coefficients do not give
5. Extension of contact terms — existence only
6. Which goal the statements belong to, and what is not said
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01

Statement — conditions on the configuration and the two bounds

Configuration. Take a centre of expansion $o \in \mathbb{R}^4$, a scale $r > 0$, a constant $M \geq 0$, and $\eta, c > 0$. Place input fields $A_1, \ldots, A_n$ (degrees d_i, total degree $w = \sum d_i$) at points x_i and spectator fields $B_1, \ldots, B_\ell$ at points y_ℓ, and impose the following Source (`ChallengeSpec.lean` in `yang-mills-group/OpenAI-YM-20261028`: `Within` lines 65–66, `Clear` lines 67–68) Assessment (`Properties.ope` in Appendix A).

$$\|x_i - o\| \leq Mr, \qquad \|x_i - x_j\| \geq \eta r \ (i \neq j), \qquad \|y_\ell - o\| \geq c, \qquad \|y_\ell - y_m\| \geq c \ (\ell \neq m$$

The input fields are gathered around o at size r without their shape collapsing (points are at least ηr apart), and the spectators are at least c away from o and from each other.

Output fields. For each degree j there is a finite set of channels, and for each channel $\alpha = (j, i)$ an output field $O_\alpha \in F_j$ (`actor`) and a coefficient $c_\alpha(A; x, o) \in \mathbb{C}$ (`coefficient`), as fields of the hierarchy T (lines 43–47).

The two bounds. The structure `OperatorProductExpansion` (lines 70–80) says the following Source. For every $\varepsilon > 0$ and all sufficiently small $r > 0$,

$$|c_\alpha(A; x, o)| \leq r^{\,d_\alpha - w - \varepsilon} \qquad \text{(configuration: the first two conditions of (1))} \ (2)$$

$$\Big| S^s\Big(\prod_i A_i(x_i)\, B\Big) - \sum_{d_\alpha \leq D} c_\alpha(A; x, o)\, S^s\big(O_\alpha(o)\, B\big) \Big| \leq r^{\,D+1-w-\varepsilon} \qquad \text{(configuration: all}$$

(3) holds for every truncation degree D, where s is the mixed theory S^{mix} or any phase S^ξ, and $B = \prod_\ell B_\ell(y_\ell)$. The assessment writes it in the form (20) with $o = 0$ and $x_i = r\,\tilde{x}_i$ Assessment (§ 7.5).

The field `ope` of the published statement is this structure (`Properties`, line 180). The construction's theorem additionally has in its conclusion a nested OPE (`Headline.NestedOPE`, `YM/Nonperturbative/HeadlineSpec.lean` lines 128–138; estimates on coefficients and remainders for clusters split in the shape of a tree) Source (`Construction.lean` line 28). The assessment writes that "under additional separation and scale conditions, nested expansions hold" Assessment.

02

How to read the powers

The powers in (2)(3) have the same form as engineering dimension counting Inferred . If a field of degree d scaled exactly as length to the power $-d$, then $\prod A_i(x_i)$ would scale as length to the $-w$, $O_\alpha(o)$ as length to the $-d_\alpha$, and the coefficient would have size $r^{d_\alpha - w}$. The remainder after summing up to degree D has the size of the term of the next degree $D+1$, r^{D+1-w}.

The arbitrarily small loss $\varepsilon > 0$ is there to absorb logarithm-like behaviour Assessment (the committee's example `CommonOPE-20261028.lean` lines 48–52). In an asymptotically free theory, the actual scaling of fields deviates from the engineering dimension by powers of $\log$ (the $\log^{-2}$ in 13-02 is an example). A power bound does not distinguish powers of $\log$, so a loss of ε is enough. The committee's example notes that the prefactor 1 is obtained by shrinking the radius, and is not an explicit value of the radius nor a computation of perturbative coefficients.

For channels whose degree j is smaller than the total input degree w (for example the identity field, $j = 0$), the power in (2) is negative, so the coefficient is allowed to grow as $r \to 0$. This is how the expansion expresses the fact that correlations become singular as two fields approach each other.

03

How the coefficients and channels are built

Appendix A of the assessment describes the construction as follows Assessment .

- **Channels** (`Channel`): split $F_d = F_{d-1} \oplus E_d$ and number a basis of E_d (each element invariant under conjugation). The number of channels of degree d is $\dim E_d$. The output field `actor` is an element of that basis.
- **Coefficients** (`coefficient`): from finitely many correlations of the input fields with additional test insertions, build a field in F_d, average it over the four-dimensional orthogonal group, symmetrise it with respect to conjugation, remove the lower-degree components, and take its coordinates in the basis $O_{(d,i)}$.

The coefficients are built only from the input fields and positions, and do not take the spectators, the state or the truncation degree as arguments. This can be read off the type Source (the type of `coefficient`, `ChallengeSpec.lean` lines 46–47: the arguments are the list of input labels, the output degree and channel, the input positions and the centre).

The committee's example shows, as checks of the dictionary, that adding the channel coordinates back recovers the coefficient field (`channel_assembly`, line 128), and that

the expansion of a single insertion returns that field itself (`single_insertion_recovered`, line 133) Assessment . The example says that this is not a computation of the values of an interacting multi-point OPE.

04

Uniformity over states, and what the common coefficients do not give

Uniformity. In the Lean form of (3), the universal quantifier over states s sits **inside** the quantifier "for sufficiently small r" ($\forall^{f}\ r$) Source (`ChallengeSpec.lean` line 76). That is, the radius is fixed before the state is asked for, and the remainder bound is uniform over the mixed theory and all phases. The committee's example makes this order explicit in terms of a radius ρ Assessment (`remainder_uniform_in_state`, line 79: "ρ is chosen before s and all admissible positions. It may depend on the labels, D and the geometric constants, but not on the phase").

Common coefficients do not give agreement of phases. Subtracting the expansions for two states s, t, since the coefficients are the same,

$$\left|(S^{s}-S^{t})\Big(\prod A_i\, B\Big)-\sum_{d_\alpha\le D} c_\alpha\,(S^{s}-S^{t})\big(O_\alpha(o)\,B\big)\right|\le 2\,r^{\,D+1-w-\varepsilon} \tag{4}$$

Proved here (triangle inequality) Assessment (`difference_of_states`, line 99). The difference is expanded in differences of correlations of the output fields, and there is no reason for those to be 0. The assessment writes that the case without spectators makes this clear: even with common coefficients, the one-point functions $S^{s}(O_\alpha(o))$ may differ from phase to phase Assessment (§ 7.5).

05

Extension of contact terms — existence only

The correlations of 8-05 are functions defined only where the points are pairwise distinct. Using the OPE, they can be extended to distributions on test functions that include coinciding points. The definition `Hierarchy.Contacts`, placed in the public namespace (it is not in the conclusion of the published theorem), has the following form Source (`ContactSpec.lean` lines 12–15).

$$\forall s,\ n,\ v\quad \exists\, D:\ C_c^{\infty}\big((\mathbb{R}^4)^n\big)\to\mathbb{C}\ \text{continuous linear},\quad \operatorname{supp} f\subset\{x_i\ /=x_j\}\ \Rightarrow\ D(f)=\int$$

That is, it says only that there **exists** a distribution that agrees with the original correlations where the points are separated. The construction provides it with `contactDistribution` (`ContactBridge.lean` line 14), and the conclusion of the

construction's theorem contains `(hierarchy F Q).Contacts` Source (`Construction.lean` line 22).

The extension cannot be unique: adding distributions supported on the diagonals (delta functions and their derivatives) does not change the values where the points are separated (4-04). The assessment writes that uniqueness of this extension, the full OS axioms (positivity) on the unrestricted space of test functions, and convergence from the lattice have not been shown Assessment (§ 7.5, § 10). The committee's independent OS check uses test functions all of whose derivatives vanish on the diagonals, so contact terms are invisible to it (8-05 § 04).

06

Which goal the statements belong to, and what is not said

Goal	What is there
The published theorem `JointExistence`	The two OPE bounds (2)(3) (field `ope`) Source . The extension of contact terms is **not included**
The construction's theorem `constructed`	In addition to (2)(3), the OPE at separated points `SeparatedOPE`, the nested OPE `NestedOPE`, and the extension of contact terms `Contacts` Source (`Construction.lean` lines 22, 28)
The committee's goals	The consistency check of the OPE statement [A8] is a **separate endpoint** from the joint goal, and passed in a separate Comparator run Assessment (Assessment § 9.3, § 11). The statement `CommonOPE` states the two bounds directly, without using the source's predicates

What is not said Assessment (Assessment § 7.5, `CommonOPE-20261028.lean` lines 159–165):

- Convergence of the expansion as $D \to \infty$.
- Uniqueness of the coefficients when the normalisation of fields or the OPE prescription is changed.
- The numerical values of the perturbative Yang–Mills coefficients (agreement with perturbation theory to all orders is unproved).
- That the coefficients do not depend on the choice of subsequence of volumes.
- Identifying phases from agreement of coefficients.
- Uniqueness of the extension of contact terms, OS positivity, convergence from the lattice.

D is a truncation by degree of fields, not a loop order Assessment .

07

What this book's investigation had seen

This book has **neither** an OPE nor contact terms. This book had no space of local fields (8-03 § 06), so there was nothing on which to set up an expansion as points approach each other.

Among this book's observables, the leading term of a small Wilson loop is an insertion of F^2 Inferred . In the language of the published construction, the expansion as the loop is shrunk to zero size is a question similar to the OPE, but loops are not in the space of local labels, so it cannot be discussed in the form (2)(3) as it stands. The fourth open question of the assessment, "construct Wilson-loop observables and relate them to local observables", points to this gap (16-02, 14-06).

08

What we have not read

- The content of the construction of the coefficients (`YM.OPE.Prescription.ope`, `YM.OPE.Orbits.averageV` and others), and the proofs of (2)(3) (`coefficient_separated`, `quantitative_separated`).
- The meaning of the tree conditions of the nested OPE (`Accept`, `Tapered`).
- How `contactDistribution` is built (whether it subtracts the singular part using the OPE and then extends).
- How the complement is chosen in the splitting by degree of output fields, $F_d = F_{d-1} \oplus E_d$.

What this chapter can say

Can say	Cannot say
Source The published statement contains the bound (2) on the OPE coefficients and the bound (3) on the remainder truncated at degree D	Convergence as $D \to \infty$; agreement with perturbative coefficients
Source The remainder bound is uniform over the mixed theory and all phases (the radius is fixed before the state)	Independence of the coefficients from the subsequence of volumes
Source The coefficients do not take the spectators, the state or D as arguments	Uniqueness of the coefficients when the normalisation or prescription is changed
Proved here From the common coefficients only the expansion (4) of the difference of two states follows, not agreement of phases	Identification of phases

Can say	Cannot say
Source The construction's theorem contains the existence (5) of an extension of contact terms	Uniqueness of the extension, OS positivity, convergence from the lattice; a statement of the extension in the published statement

Sources and reproduction

Item	Kind	Source / tool
OPE statement, configuration, type of the coefficients	Source	`ChallengeSpec.lean` lines 43–80 and 177–181 of `yang-mills-group/OpenAI-YM-20261028` (Apache-2.0, revision 3456e5e)
Contact terms, nested OPE, the construction's conclusion	Source	`ContactSpec.lean` lines 12–15, `ContactBridge.lean` line 14, `YM/Nonperturbative/HeadlineSpec.lean` lines 128–138, `Construction.lean` lines 15–31
Equation (20), how channels and coefficients are built, what is not said, the separate endpoint	Assessment	The assessment, `yang-mills-group/YM` `docs/assessment-1.pdf` (2026-11-10), § 7.5, § 9.3, § 10, § 11, Appendix A (`Channel`, `actor`, `coefficient`, `Properties.ope`)
The committee's OPE example	Assessment	On the assessment side, `submissions/openai-20261028/docs/examples/CommonOPE-20261028.lean` in `yang-mills-group/YM`, revision 55ba9fc, lines 48–52, 79, 99, 128, 133, 159–165
Arbitrariness of contact terms	Cited	Introduction in 4-04

Terms used in this chapter — operator product expansion, channel, spectator, remainder bound, uniformity over states, extension of contact terms. Their meanings are in the A-02 glossary.

Next chapter: 14-01 Comparing skeletons — in Part 14 we compare the published construction with this book's Lean, stage by stage.