Volume 2 Existence — reading the published construction

Part 14 Comparison — this book's Lean, and ideas this book's study lacked

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Comparing skeletons — a table stage by stage

> This chapter is this book's own investigation. Descriptions of the published construction are established knowledge (a reading of the published construction).

What this chapter shows — We split the proof of the published construction into eleven stages, from the lattice and reflection positivity to the mass gap and Wilson loops, and lay out in one table, stage by stage, "what the published construction does" and "what the Lean of this book's study had". We then separate what this book's study had seen (fix the volume → compactness → carry properties to the limit; fix the coupling by matching; blocks of odd side placed at the centre) from what it had not seen (the core of the multi-step estimates; everything from local fields onward).

Prerequisite chapters — 6-03 the skeleton of the proof, 28-01 implication of subsequential limits, 19-07 statements to be shown.

Said first — The results of the published construction are written at the level "the committee's assessment is that it is proved in Lean". What this book's study checked in the source goes only as far as the definitions and the form of the statements; there are many places where we have not read the content of the proofs. "Present" on this book's side means either a theorem at one scale with fixed lattice spacing, or an implication with all its hypothesis fields shown. It does not mean that this book's study showed anything about the continuum limit.

The order of this chapter

1. Conventions of the comparison — two levels, and five words for status
2. Table by stage (Table 1) — eleven stages
3. Lattice, reflection positivity, chessboard — present; the differences are the root seminorm and rarity
4. One renormalisation step and matching the coupling — partial; design and definitions differ
5. Multi-step estimates and large fields — absent; the part that is about 70% of the code
6. The ultraviolet limit at fixed volume — skeleton only; subsequences and the full sequence

7. Infinite volume, reconstruction, local fields — what is present in another regime, and what is absent
8. What was seen and what was not — three and two
9. What we have not read — with locations

Labels: Source = we read the source of the published construction; Inferred = this book's inference; Assessment = from the committee's assessment. The other labels (Proved here, Cited, Reading, Lean and so on) mean the same as in the rest of the book (details in "Labels" in 6-00). For Lean theorems of this book's study, the name and scope are written on the same line, outside the labels.

01

Conventions of the comparison

This part sets side by side this book's study and the existence construction for pure Yang–Mills that was submitted on 28 October 2026 and published on 10 November together with the committee's assessment (below, "the published construction"). The code of the published construction is in the repository `yang-mills-group/OpenAI-YM-20261028` (Apache-2.0), and the committee's assessment is `docs/assessment-1.pdf` (2026-11-10) in `yang-mills-group/YM`.

The two sides stand at different levels of certainty.

- **The published construction**: the results stand at the level "the committee's assessment is that it is proved in Lean" Assessment . What this book's study checked itself goes only as far as reading the definitions and the form of the statements in the source Source .
- **This book's study**: theorems machine-checked in Lean, or implications with all hypothesis fields shown. Every theorem is either a statement at one scale with fixed lattice spacing or an abstract implication, and none says anything about the continuum limit.

The "Status" column of Table 1 uses the following five words.

Word	Meaning
Present	A statement of the same kind is also a theorem in the Lean of this book's study

Word	Meaning
Partial	The objects or the shape of the skeleton are there, but definitions, design or strength differ; or it is there only in the form of an implication
Absent	Not in this book's study, neither as a theorem nor as an implication
Present in another regime	A statement of the same kind is in this book's study, in a different coupling regime (the strong-coupling window)
Only in this book	A kind of statement not among the claims of the published construction is in this book's study

02

Table by stage

The proof of the published construction is assembled in the following order: starting from the calendar (the choice of constants), the one-step kernel, the representation of the density at each step and the induction, the choice of coupling by shooting, the ultraviolet limit at fixed volume, infinite volume, Euclidean invariance and phase decomposition, reconstruction, and the evidence that it is Yang–Mills (details in 6-03) Source . We split this into eleven stages.

Table 1 Comparison by stage (as of 2026-11-13)

Stage	The published construction	This book's study	Status
Lattice and reflection positivity	Planes through lattice points and planes through midpoints of edges, all four axes, the Wilson measure with $\beta \geq 0$. Functions are gauge invariant and local to an open slab not containing points on the mirror (for midpoint planes, $r <$ half) Source	The same two kinds of plane. For lattice-point planes, all real β, all four directions, functions on the closed positive half (gauge invariance not imposed). For midpoint planes, $\beta \geq 0$ and gauge-invariant functions (20-01)	Present (different class of functions)
Chessboard	The chessboard inequality and the triangle inequality for its root. Used as a seminorm measuring the size of observables Source	The Chessboard Estimate, and rarity of large plaquettes at weak coupling (uniform in volume, 22-01). No form as a seminorm	Present
One renormalisation step	A probability kernel. Blocks of odd side placed at the centre of a coarse point, the predictor projected onto the group, gauge fixing in two strands Source	A deterministic push-forward map (side 2, overlapping weights), and the side-3, centred, uniform design (14-02)	Partial

Stage	The published construction	This book's study	Status
Matching the coupling	Match a coefficient in the representation to a reference value. Shooting method. A running law Source	A definition that matches with a quantity fixed by the measure alone. Existence of a matched sequence holds unconditionally only for the side-3 design (`Block3.tuned3_exists`) and for the vortex ratio on each lattice (`ThetaLimit.tuning_all`). In the current design (side 2) existence itself is conditional (if the value at the lower end is at most the target), and the chain theorems that use a matched sequence are conditional on "if it exists". No running law (14-03)	Partial
Multi-step small-field estimates	Induction hypothesis and the one-step theorem Source . About 70% of the code Inferred	One step of the free field (15-12), and implications with the estimates as hypotheses (28-01, 28-02)	Absent
Large fields	Re-count seeds at each step and set up an inequality per seed for every configuration satisfying the gauge section and the cell conditions Source (14-04)	Only rarity at one scale. Treatment across steps remains a premise (the large-field estimate in 28-04)	Absent
Ultraviolet limit at fixed volume	Correlations of local fields converge along the full sequence of spacings Assessment . In the published theorem too, the full sequence is used for the cutoff Source	Subsequential limits of the push-forward measures on the coarse lattice. The estimates are hypothesis fields (28-01)	Partial (skeleton only)
Infinite volume	Subsequence (Arzelà–Ascoli) Assessment	Inside the strong-coupling window, a limit without subsequences (20-03)	Present in another regime
Euclidean invariance, phase decomposition, reconstruction	Present Assessment	The lattice reconstruction vessel, and the mass gap at strong coupling (20-04). No Euclidean invariance or phase decomposition	Absent (the vessel is present in another regime)
Local fields, stress tensor, operator product expansion	Present Assessment	Not treated	Absent
Mass gap, Wilson loops, vortices	No statement of exponential decay. No continuum Wilson loops. Only periodic boundary conditions Source	Mass gap and area law at strong coupling, the vortex floor, pairs of Polyakov loops (14-06)	Only in this book

By line count, the code of the published construction is roughly 70% multi-step induction and estimates, roughly 10% local fields and expansions, roughly 10% the ultraviolet at fixed volume, and about 4% reconstruction, with the definitional layers together under 4% Inferred . Of the 250,319 declarations, 100,168, including the headline theorems themselves, are reached from one of the seventeen headline theorems listed in the verification records Source (the same number as in Assessment § 9.1). The two rows this book's study marked "absent" (multi-step small-field estimates and large fields) also make up most of the construction by volume.

What Table 1 does not say. "Present" means that a statement of the same kind is also in this book's Lean, not that a statement of the same strength, the same class of functions and the same coupling regime is there. The "only in this book" row, too, consists of statements at fixed lattice spacing and says nothing about a continuum theory.

03

Lattice, reflection positivity, chessboard

Reflection positivity. The published construction has reflection positivity with $\beta \geq 0$ for the Wilson measure on the periodic torus, for both planes through lattice points and planes through midpoints of edges, along all four axes (`reflected_positive` in `YM/Slice/Reflection.lean:98`) Source . The class of functions is gauge-invariant, measurable and bounded, local to the open slab $0 < r <$ half not containing points on the mirror (or the opposite face) for planes through lattice points, and to $r <$ half for planes through midpoints (`Strict` in `YM/Cells/Cut.lean:30`–:33, `Interior` and `Positive` at :109–:118) Source . The Lean of this book's study has the lattice-point-plane form `TorusRP.wilson_rp` (all real β, bounded measurable functions on the closed positive half, gauge invariance not imposed), the four-direction form `PeierlsPerm.wilson_rp_dir`, and the middle-plane form `MidRP.wilson_rp_mid` ($\beta \geq 0$, gauge-invariant functions) (20-01). Both sides are formalisations of known theorems Cited . Since both have the same two kinds of plane we mark it "present", but the classes of functions differ.

Chessboard. The published construction adds to the chessboard inequality (`tiles` in `YM/Cells/Chess.lean:80`) the triangle inequality for its root (`adds`, same file :108) Source . The quantity obtained by raising the expectation of the product of box-wise copies to the power of the reciprocal of the number of boxes (`rootTest` in `YM/Cells/Tiling.lean:226`) is used as a seminorm that measures the size of observables independently of the volume Source . The statement `Wcmain` (`YM/Extraction/Center.lean:38`), which states the ultraviolet estimates in this seminorm, is one of the inputs to compactness in infinite volume Source . It is not the

only input from the ultraviolet (for example `MergeProof.Accent.rated`, `YM/MergeProof/Powder.lean:48`, takes `Wcmain` and `Scrim` together)Source .

The Lean of this book's study has the Chessboard Estimate `FoldWilson.wilson_chessboard_even` (even side, all real β) and, using it, rarity of large plaquettes at weak coupling `FoldCap.rarity_explicit` (uniform in volume, explicit threshold) (22-01). There are two differences. This book's side lacks the triangle inequality for the root (the form as a seminorm), and on the published construction's side, within what we read, we did not find a rarity statement that uses the chessboard for probabilities of eventsSource (within the range searched).

04

One renormalisation step and matching the coupling

One step of the published construction is a probability kernel that builds a "prediction" of the coarse links from the fine configuration and draws the coarse links with Gaussian-type fluctuations around it (`density` in `YM/RG/Block.lean:71`)Source . One step of this book's study is a deterministic map that projects a weighted average of path products back onto the sphere, and the measure is pushed forward by that map (19-04). The side-3, centred, uniform design is close in form to the published construction's "odd side, placed at the centre, weights from convolving uniform distributions", and reflection positivity of the measure pushed forward with that design is in Lean. Details are in 14-02.

For the coupling, both are of the type "fix the scale by matching a quantity determined at unit length to a fixed value"Inferred . The differences are the quantity matched (in this book, a quantity determined by the measure alone; in the published construction, a coefficient in the representation of the density) and how existence is derived (in this book, global endpoints and the intermediate value theorem; in the published construction, nested intervals of bands at each step). Details are in 14-03. In both rows the objects or skeleton are there but the definitions and design differ, so we mark them "partial".

05

Multi-step estimates and large fields

The published construction represents the effective density at each step m in a fixed form (truncated action, small-field activities, a gas of large-field activities), and carries that form and its estimates through all steps as the induction hypothesis `History.Conv` (`YM/RG/History.lean:213`)Source . Steps are linked by push-forward identities (`History.Transfer`, same file :223), and the statement for all steps is `History.Selected`

(same file :234) Source . The form of the one-step theorem is the field `advances` of `Directed.Program` (a structure in `YM/PCalc/Band.lean:65`), and the one-step certificate is `Move` (same file :49) Source .

This book's study has nothing corresponding to this as a theorem. What it has is one step of the free field (the covariance of the linearised block map, 15-12, 15-13) and implications with the estimates as hypothesis fields (28-01, 28-02). Large fields went only as far as rarity at one scale (22-01), and estimates across steps remained, without a settled form, as the "large-field estimate" and "large-field gas" in the list of premises (28-04) (28-06). These two rows are "absent". How the published construction handles them is compared in 14-04.

06

The ultraviolet limit at fixed volume

The published construction fixes the physical volume and shows that correlations of local fields converge along the full sequence of spacings Assessment (Assessment § 4.3). The location in Lean is `Campaign.realUV` (`YM/FullVolume/Fields/Filled.lean:45`), and the chain to the main theorem has been traced Source . In the convergence field `Converges` (`PublicWilson.lean:85-90`) of the published theorem `JointExistence` (`ChallengeSpec.lean:186`) too, convergence along the full sequence is required for the cutoff at each volume, and only the volume side is a subsequence Source . The stronger form with kernels and differences (`PositionsUV … Sequence.full`) is only in the conclusion of the construction's theorem `constructed` (`Construction.lean:15`) Source .

The skeleton is this Source . Show that the effective density at a fixed coarse step j is Cauchy uniformly in the cutoff N (`Flux.fanned`, `YM/FullVolume/Fields/Cement.lean:50`); bound the difference between the true correlations and the read-out at step j by a quantity independent of N (`Sever.echoes`, `YM/FullVolume/Fields/Peat.lean:42`); and from this, convergence at the fixed step and the triangle inequality, conclude that the true lattice correlations form a Cauchy sequence in N (`Rising.arrives`, `YM/FullVolume/Fields/Timber.lean:74`).

On the side of this book's study, Lean has the implication that subsequential limits of the push-forward measures $\nu_{n,m}$ exist and that properties such as reflection positivity pass to those limits (`BoxWalk.chain_end_to_end_box`, 28-01). Subsequential limits exist for any family of probability measures, so the content of this implication is "carrying properties over". The estimates on the limit are all hypothesis fields, and include quantities not defined from the lattice. The shape of the skeleton is close, but the content of the estimates and full-sequence convergence are missing, so we mark it "partial (skeleton only)".

07

Infinite volume, reconstruction, local fields

Infinite volume. Taking the volume-uniform ultraviolet estimates as input, the published construction derives volume-uniform boundedness and equicontinuity from reflection positivity on the original lattice, the chessboard and the trace representation of the transfer operator, and takes a subsequence by Arzelà–Ascoli (`first_surface`, `YM/Extraction/Lipschitz.lean:166`; `crops`, `YM/Extraction/Held.lean:136`) Source . No material for uniqueness is included Assessment (Assessment § 5, § 8). The Lean of this book's study has, inside the strong-coupling window ($|\beta|$ small), an infinite-volume limit that converges to a single measure along the sequence of sizes itself (`InfVol.wilson_infvol_unique`, 20-03). The coupling regime is different, so it is "present in another regime".

Euclidean invariance, phase decomposition, reconstruction. The published construction lifts rotational invariance from the Ward identities of the stress tensor, splits into phases by the Gelfand representation of a commutative algebra of operators compressed to the vacuum space (`OS.Selection`, `YM/Phase/Mixture.lean:127`), and reconstructs phase by phase Assessment Source . This book's study has the reconstruction vessel of the lattice transfer operator and the mass gap in the strong-coupling window (`InfMain2.wilson_infvol_main2` and others, 20-04). It has no continuum Euclidean invariance and no phase decomposition.

Local fields. The published construction has labels for local fields, renormalised insertions, the stress tensor, the trace anomaly and the operator product expansion (Parts 8 and 13) Assessment . This book's study does not treat these.

08

What was seen and what was not

We reread Table 1 in terms of whether the shape of the skeleton had been seen Inferred .

What was seen (three things).

1. **Fix the volume → compactness → carry properties to the limit.** The implication of subsequential limits of this book's study (28-01) corresponds to the published construction's type "put the estimates as hypotheses and conclude a limit" (`FullVolume.positioned`, `YM/FullVolume/Profiles.lean:42`) and to its type "carry properties to the limiting table under conditions" (`realPositive`, `YM/Extraction/Harvest.lean:46`) Source . The order — fix the volume first, then go to infinite volume — is also the same.
2. **Fix the coupling by matching.** The type "choose the bare coupling so that a quantity at some coarse step takes a fixed value" is the same. This book's side has a

definition that matches with a quantity determined by the measure alone. Existence of a matched sequence is in Lean unconditionally only for the side-3 design (`Block3.tuned3_exists`) and for the vortex ratio on each lattice (`ThetaLimit.tuning_all`); in the current design (side 2) existence itself is conditional, and the chain theorems that use a matched sequence are conditional on "if it exists" (14-02 § 06, 14-03).

3. **Blocks of odd side placed at the centre.** The side-3, centred, uniform design points in the same direction as the way the published construction places its blocks (14-02).

What was not seen (two things).

1. **The core of the multi-step estimates.** Carrying the representation of the density at each step by induction; re-counting large fields at each step and setting up an inequality per seed for every configuration satisfying the gauge section and the cell conditions; putting a weak ledger in the induction hypothesis while having the one-step certificate produce strong bounds; and discharging the step-uniform conditions on the calendar side (14-04, 14-05).
2. **Everything from local fields onward.** Calibration of observables across steps, Cauchy at a fixed step and the one step that removes the step, the stress tensor and Euclidean invariance, phase decomposition. The endpoint of this book's study was the push-forward measure on the coarse lattice, not correlations of local fields. With a different endpoint, the skeleton needs two extra layers (calibration and the step-removing step)Inferred .

Conversely, this book's study has kinds of statements not among the claims of the published construction (the mass gap, Wilson loops and the area law, the vortex free energy). All are formalisations of known types at fixed lattice spacing, statements at the opposite end (strong coupling) from the questions the published construction left open (14-06).

09

What we have not read

- The content of `Mallow.faces`, where the proof of the one-step theorem lives (`YM/PhysicalNext/RG_NEXT_COMPLETE_REPEATABLE_NONLINEAR_STEP/Mallow/Pediment.lean`).
- The record of how declarations are used lists only the first headline found by depth-first search. The chains to the main theorem (`jointExistence`, `Solution.lean:4`) that this book's study checked are limited to the paths `rodB` → `constructionRun` and `realUV` → `fromFunded`, the path `allB` for convergence in the cutoff, `sailsW`,

and a few others. We have not checked whether the other declarations in Table 1 are among the dependencies of the main theorem.

- The theorem that builds the phase decomposition, and how the selected phase is chosen.
- We have not matched the Lean of this book's study against the Lean of the published construction, statement by statement in wording, for the same objects (because the types of the objects differ).

What this chapter can say

Can say	Cannot say
Source The locations of the declarations for each stage of the published construction (in the text and the table of sources), checked against lines of the source	The content of their proofs. The dependency chain of the main theorem is traced only for some declarations
Assessment The published construction takes the ultraviolet limit at fixed volume along the full sequence of spacings, and the infinite-volume limit along a subsequence (Assessment § 4.3, § 5)	Uniqueness of the infinite-volume limit, the mass gap (listed as open questions in Assessment § 8)
The Lean of this book's study: reflection positivity for lattice-point planes and middle planes, the Chessboard Estimate, the infinite-volume limit without subsequences and the mass gap in the strong-coupling window (Part 20)	Varying the lattice spacing, the continuum limit, outside the window
The Lean of this book's study: the implication of subsequential limits (28-01). A conditional theorem with all hypothesis fields shown	None of the hypothesis fields is shown on the four-dimensional lattice
Inferred What this book's study had seen were three corners; what it had not seen were the core of the multi-step estimates and everything from local fields onward	"Seen" means that the shape of the skeleton is close, not that we had the same estimates

Sources and reproduction

Item	Kind	Source / tool
Code of the published construction	Source	Repository `yang-mills-group/OpenAI-YM-20261028` (Apache-2.0, revision 3456e5e). Paths are relative to the repository root. Declaration names and lines are given throughout the text
The committee's assessment	Assessment	`docs/assessment-1.pdf` (2026-11-10) in the repository `yang-mills-group/YM`. § 2, § 4.3, § 5, § 6, § 8
How declarations are used	Source	`verification/declaration-usage.tsv` in the same repository. "used" means reached from one of the seventeen headline theorems listed in the verification records, not used by the published theorem

Item	Kind	Source / tool
Distribution of line counts	Inferred	A rough count by directory (this book's study's own tally)
Theorems of this book's study	Lean	The tables of sources in Parts 19–22 and Part 28

Terms used in this chapter — published construction, the committee's assessment, probability kernel, push-forward measure, reflection positivity, root seminorm of the chessboard, matching, shooting method, induction hypothesis, ultraviolet limit at fixed volume, implication of subsequential limits, phase decomposition. Their meanings are in the A-02 glossary.

Next chapter: 14-02 Comparing one-step designs — we compare the "one renormalisation step" row of Table 1 in detail across three designs.

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Comparing one-step designs — the push-forward map and the probability kernel

> This chapter is this book's own investigation. The description of the published construction (§ 04) is established knowledge (a reading of the published construction).

What this chapter shows — We lay out in one table three designs for one renormalisation step: the current design of this book's study (side 2, overlapping weights, a deterministic push-forward map), the side-3, centred, uniform design, and the design of the published construction (a probability kernel that draws the coarse links with Gaussian-type fluctuations around a predictor). We state what was closed in Lean for the side-3 design (reflection positivity of the pushed-forward measure, existence of a matched sequence), and explain why this is needed nowhere in the skeleton of the published construction (what is taken to the limit are correlations under the measure of the original lattice, and reflection positivity is used only on the original lattice).

Prerequisite chapters — 19-04 The block map and one step of renormalisation, 9-01 the block kernel, 14-01 Comparing skeletons.

Said first — We read the definition of the published construction's one-step kernel in the source `Source` , but not the content of the proofs of gauge fixing (`Branch.align` , `Axial.existence`). The theorems for the side-3 design are statements on a finite lattice with fixed lattice spacing, and this design is not the main line of this book's study. This chapter does not claim which design is better suited to the continuum limit.

The order of this chapter

1. What one step has to decide — the rule, and how the measure is carried
2. This book's current design — side 2, overlapping weights — a deterministic map and push-forward
3. This book's side-3 design — centred, uniform — commutes with reflections without gauge transformations
4. The published construction's design — a probability kernel — predictor, projection onto the group, gauge fixing in two strands, growing sides

5. Table of the three designs (Table 1) — nine items
6. What was closed in Lean for the side-3 design — reflection positivity of the pushed-forward measure, and existence of a matched sequence
7. Why it is not needed in the skeleton of the published construction — a difference in what is taken to the limit
8. What the choice of design affected — this book's skeleton and the published construction's skeleton
9. What we have not read

Labels: Source = we read the source of the published construction; Inferred = this book's inference; Assessment = from the committee's assessment. The other labels (Proved here, Cited, Reading, Lean and so on) mean the same as in the rest of the book (details in "Labels" in 6-00). For Lean theorems of this book's study, the name and scope are written on the same line, outside the labels.

01

What one step has to decide

One renormalisation step is a pair: a rule that produces a coarse-lattice configuration V from a fine-lattice configuration U, and a way of carrying the fine-lattice measure to the coarse lattice (introduction in 7-05). There are two types of rule.

- **Deterministic map**: $V = B(U)$. The measure is carried by push-forward $B_*\mu$ ($B_*\mu(E) = \mu(B^{-1}E)$).
- **Probability kernel**: the conditional distribution $T(U, \mathrm{d}V)$ of V given U. The measure is carried by $\int T(U,\cdot)\,\mu(\mathrm{d}U)$. A deterministic map is the case of a kernel concentrated on a point mass, $T(U,\cdot) = \delta_{B(U)}$.

Either way, five things must be decided: the side of the block (by what factor the lattice spacing is reduced), where on the fine lattice the coarse points sit, the fine paths that build a coarse link and their weights, the procedure that brings an average that has left the group back onto the group, and compatibility with the lattice symmetries and gauge transformations.

02

This book's current design — side 2, overlapping weights

The map on the main line of this book's study was defined in 19-04. The side is 2, and coarse points sit on the fine lattice points $2x$. The value of a coarse link b is obtained by choosing, for each transverse direction, a shift $-1, 0, +1$ with weights $1/8,\ 3/4,\ 1/8$, averaging the products over the family of paths split equally among the six orders,

$$q_b(U) = \sum_{o,A} w(o,A)\, U(\gamma_{o,A}(b)) \quad (1)$$

and bringing it back onto the unit sphere by the retract with threshold r_0, $\rho_{r_0}(q) = q/\max(|q|, r_0)$. The measure is carried by the deterministic push-forward $\nu = B_*\mu$. The map does not depend on the coupling β.

Because paths shifted sideways are averaged, neighbouring blocks share fine links (an overlapping average). Reflection positivity of the pushed-forward measure is a theorem in this design for functions in the class of slabs of restricted width (`BoxWalk.wilson_h0p`, 28-05).

03

This book's side-3 design — centred, uniform

In the side-3 design, on the lattice $(\mathbb{Z}/3^n L)^4$, coarse points are placed at the centres of blocks, and a coarse link is built as the uniform average over a family of paths of the form "outline + three straight steps in the direction of the coarse link + return along the outline" (`Block3.paths`, `Block3.wts`; 1944 indices, 633 distinct paths, weights summing to 1). The procedure for bringing the average back onto the sphere is the same as in the current design.

With centring and uniform averaging, the map commutes directly with reflection in a lattice-point plane, without any gauge transformation (`Block3.blockMap_refl_cfg`; for all steps, `Block3.down_refl`; with sphere values). Flipping the sign of the fine links that cross a cross-section of a block flips the sign of exactly one coarse link (one step: `Block3.blockMap_flip`; the set form for all steps: `Block3.down_flipSet`). The commutation properties were also checked numerically, with code that copies the Lean definition of the map literally, in the non-abelian, four-dimensional, two-step case Reading . This design is not the main line of this book's study; it was examined as a parallel track.

04

The published construction's design — a probability kernel

One step of the published construction is a probability kernel (`StepData.density` in `YM/RG/Block.lean:71`) Source . As a formula, for a fine configuration U and a coarse configuration V,

$$T_\beta(U,V)=\sum_t \ell(U,t)\prod_b \frac{1}{Z_\beta}\exp\!\left(-\beta\, c_{\mathrm{pen}}\,\mathrm{price}\!\left(V_b\,Q_b(U,t)^{-1}\right)\right) \tag{2}$$

which is a density in V with respect to the product of Haar measures Source . Here t is the label of the choice (the gauge frame and the predictor label), $\ell(U,t)$ its probability (`likelihood`), $Q_b(U,t)$ the predictor for the coarse link b, $c_{\mathrm{pen}}>0$ the penalty coefficient (`kernel.penalty` in the source's wording), and price a function measuring the distance of a group element from the identity. The coarse links are not constrained to the predictor but drawn with Gaussian-type fluctuations around it. c_{pen} may be any positive number, and the kernel depends on β Source . Densities at successive steps are linked by push-forward with this kernel (`push`, same file :77) Source .

Side and placement. The side b_m is an odd number at least 3 chosen at each step (`Schedule` in `YM/Lattice/Scale.lean:28`; only oddness and being at least 3 are here) Source . Non-decrease is the calendar field `Fits.monotoneL`, and $\log b_m/\log D_m\to 0$ is the field `Fits.mesh` (= `Schedule.Gentle`; `YM/RG/History.lean:97`–:100, `YM/Lattice/Scale.lean:129`) Source . Together with the calendar conditions, $b_m\to\infty$ follows Inferred (derived from the definitions; growing sides; the reason is in 10-05). A coarse point x is placed at the fine point $b\cdot x$ (`embed` in `YM/Lattice/Subdivision.lean:29`), and the block is centred on the coarse point Source .

Weights. The weights on the points of a patch are the J-fold convolution of the centred uniform distribution of side b (`weight` in `YM/RG/Patch.lean:87`, summing to 1 at :90), with $J\ge 4$ growing with the step Source . The support radius is $J(b-1)/2$, overlapping the neighbouring blocks Source .

Predictor and projection onto the group. Corresponding points of two neighbouring patches are joined by parallel straight lines of length b, the transports along them are taken, and they are gauge-fixed by the frames at both ends. The group element nearest to their weighted average is the predictor (`bondCost`, `best`, `YM/RG/Patch.lean:159`, :169) Source . That the nearest element coincides with the minimiser of the weighted sum of prices follows from the identity

$$2\,d\sum_i w_i\,\mathrm{price}(f_i u^{-1})=\sum_i w_i\|f_i\|^2-\|m\|^2+\|m-u\|^2,\qquad m=\sum_i w_i f_i \tag{3}$$

(d the dimension of the representation) (`min_iff` in `YM/Branch/Center.lean:72`) Source . When the minimum (the weighted spread plus the squared distance between the average and the group, divided by $2d$) is at least the threshold `predictor_cut`, one straight line is drawn with probability given by the weights and its transport is used (`usesBest`, same file :171) Source .

Gauge fixing in two strands. The frame of a patch (the gauge fixing) is, for small fields, a symmetric choice minimising the average price over all pairs of points in the patch along coordinate paths in axis order, and for large fields, an axial gauge with the root and axis order drawn uniformly; the kernel sums over the labels satisfying the conditions (`YM/RG/Patch.lean:102`–:155) Source .

Symmetry. The kernel built this way is exactly invariant, through the sum over choice labels, under the hypercubic group (the 384 signed permutations) and coarse translations, and covariant under gauge transformations (`StepData.channel` in `YM/Branch/Covariant.lean:50` and others) Source . Lattice symmetries pull back as $x \mapsto s \cdot x + b$ (`affine`, `YM/Lattice/Path.lean:145`), not only as rotations about a coarse point. Combining the flip of axis i with $b = e_i$ gives $x_i \mapsto 1 - x_i$, so along with reflections in planes through lattice points, reflections in planes through the midpoints of coarse links are also included Source . This is covariance as a symmetry, not a statement of reflection positivity. The assessment states that gauge fixing invariant under the discrete lattice symmetries (§ 4.3) is built by "averaging over symmetry-breaking choices" Assessment (Assessment § 6.3).

05

Table of the three designs

Table 1 One-step designs

Item	This book's current design	This book's side-3 design	The published construction
Type of rule	Deterministic map, push-forward	Deterministic map, push-forward	Probability kernel (depends on β)
Side	2	3	Odd, at least 3, non-decreasing, growing with the step
Placement of coarse points	Fine lattice points $2x$ (corners of blocks)	Centres of blocks	Centres of blocks ($b \cdot x$)
Weights	Transverse shifts $1/8$, $3/4$, $1/8$, split equally among six orders	Uniform	J-fold convolution of a centred uniform distribution ($J \geq 4$, growing with the step)

Item	This book's current design	This book's side-3 design	The published construction
Paths	A family of paths that shift sideways, go two steps and come back	Outline + three straight steps + return along the outline	Parallel straight lines joining corresponding points of two patches, gauge-fixed by frames at both ends
After averaging	Back onto the sphere (retract, threshold r_0)	Same	Projection onto the group; if the spread is large, one line drawn at random
Constraint	Push-forward (the coarse link is the value of the map itself)	Same	Gaussian-type penalty around the predictor, with arbitrary coefficient $c_{\mathrm{pen}} > 0$
How symmetry is kept	Symmetric averaging over the family of paths	Centred and uniform; commutes with reflections without gauge transformations	Exactly invariant under the hypercubic group and coarse translations, through the sum over choice labels
Large fields	Outside the map (treated separately as events below the threshold)	Same	Enter as branches inside the definition of the kernel (axial gauge, one line drawn at random)

The side-3 design is close to the published construction in three respects — odd side, centring, uniform (weights with $J = 1$) — and differs in four: being a probability kernel, projection onto the group, gauge fixing in two strands, and growing sides.

06

What was closed in Lean for the side-3 design

Reflection positivity of the pushed-forward measure. Let ν be the measure obtained by pushing forward the SU(2) Wilson measure on the periodic lattice $(\mathbb{Z}/3^n L)^4$ with L even (any real β, normalised or not) by the composition of $m \le n$ steps of the side-3 map. Then for the reflection Θ in a lattice-point plane ($y_0 \mapsto -y_0$) and every bounded measurable function F depending only on coarse links lying strictly between the two planes 0 and $M_c = 3^m L/2$ whose ends are coarse lattice points (gauge invariance not imposed),

$$\int F(\Theta V)\, F(V)\, \nu(\mathrm{d}V) \;\ge\; 0 \qquad (4)$$

holds. Lean: `Block3.rp_push`, `Block3.rp_push_prob`. The proof is a short argument that reduces to reflection positivity of the fine lattice (`TorusRP.wilson_rp`, 20-01), using that the map commutes with the reflection without gauge transformations (§ 03) and the range of fine links that a coarse link uses (`Block3.down_local`). What it does not say: the version for planes through link midpoints, the closed class including links on the plane, infinite volume, the mass gap.

Existence of a matched sequence. In the side-3 design, under the push-forward at $\beta = 0$ (independent Haar), the expectation of the average real part of coarse plaquettes is 0 at every step (`Block3.Y3_zero`; because flipping the sign on a cross-section turns over exactly one coarse link and preserves Haar). If $0 < r_0 \le 1$, this expectation tends to 1 as $\beta \to \infty$ (`Block3.tendsto_Y3_one`). By the intermediate value theorem, under $0 < r_0 \le 1$, $3^{m_0} L \ge 2$ and $0 \le \kappa_0 < 1$, there is a sequence of non-negative couplings such that for every k "the expectation of the average real part of coarse plaquettes pushed forward to step m_0 is κ_0" (`Block3.tuned3_exists`; for each k one can take the largest solution). What it does not say: monotonicity, divergence or boundedness of the sequence of couplings, uniqueness of the solution. At $r_0 = 1$ the pull-back is the identity and a coarse plaquette is a product of sphere values, not an SU(2) Wilson loop. In the current design (side 2) the value at $\beta = 0$ is not 0 (numerically 0.006–0.024 Reading), and this existence was conditional on "if the value at the lower end is at most the target". The comparison of matchings is in 14-03.

07

Why it is not needed in the skeleton of the published construction

What the published construction takes to the limit are correlations of renormalised local-field insertions under the Wilson measure $\mu^{\mathrm{W}}_{N,p}$ of the original lattice. In the assessment's formula,

$$C_{N,p}(v;f) = a_N^{4p} \sum_{x_1,\dots,x_n} f(a_N x_1,\dots,a_N x_n)\, \mathbb{E}_{\mu^{\mathrm{W}}_{N,p}}\Big[\prod_{j=1}^{n} I_{N,p}(v_j, x_j; U)\Big] \quad (5)$$

and the claim is that the limit $N \to \infty$ exists along the full sequence of spacings (Theorem 4.1) Assessment (Assessment § 4.3). The renormalisation step (equation (2)) is used as a tool for estimating this expectation, and does not appear as an object of the limit Source. The pushed-forward densities at a fixed step converge (`Flux.fanned`), but the limiting measure is not defined as a named object Source (within the range of a search by name). That measure would presumably follow from the uniform convergence of the densities at a fixed step Inferred.

Reflection positivity, too, is stated only for the Wilson measure of the original lattice (`law` in `YM/Wilson/Law.lean:87`) (`reflected_positive`) Source. Reflection positivity on the continuum side is obtained by keeping non-negativity of the lattice Gram matrices in the limit of correlations (`realPositive`, `YM/Extraction/Harvest.lean:46`) Source. Within what we read, there is no statement of reflection positivity for the one-step kernel Source (within the range searched). Since the patches of the kernel straddle the reflection plane, we infer that such a statement would not be trivial Inferred.

So reflection positivity of the pushed-forward measure (the type of (4)) has no place to enter at any stage of the published construction's skeleton Inferred . In the skeleton of this book's study the situation was different. The implication of subsequential limits (28-01) takes the limit of the push-forward measures $\nu_{n,m}$ themselves and carries reflection positivity over to that limit, so "$\nu_{n,m}$ is reflection positive" is among its hypothesis fields. The theorem for the side-3 design was meant to discharge that field. With a different endpoint (what one takes the limit of), whether the same property is needed or not switches around.

08

What the choice of design affected

This book's side. What depended on the choices of the current design (weights, factor 2, threshold r_0) were the constants of the free-field single step (15-12) and the linearisation (15-13) (Table 2 in 28-05). The theorems of Parts 20–22 use only the fine-lattice measure, so they do not depend on the design. Switching to the side-3 design makes reflection positivity of the pushed-forward measure a theorem on the whole open positive side, and reduces the hypotheses for existence of a matched sequence to the design values and arithmetic ($0 < r_0 \le 1$, $3^{m_0} L \ge 2$, $0 \le \kappa_0 < 1$). The cost is rebuilding the free-field layer.

The published construction's side. With a probability kernel, the penalty coefficient c_{pen} enters the precision of the free field of one step in the form $K + c_{\mathrm{pen}} Q^{\mathsf{T}} Q$, and the constraint is no longer hard Source (11-09). Since the kernel depends on $\beta(N, p, m+1)$, the published construction's "push-forward measure" is an object whose map changes with the cutoff N, and is not the same object as the push-forward by a deterministic map in this book's study Source . Convergence of the published construction's densities at a fixed step does not give convergence of this book's push-forward measures.

What they share. Both designs aim to keep the discrete symmetries of the lattice intact after one step. This book does it by symmetric averaging over a family of paths, and the published construction by drawing symmetry-breaking choices at random and summing over them Inferred .

09

What we have not read

- The content of the proof of existence and uniqueness of small-field gauge fixing, `Branch.align` (`YM/Branch/Synchronize.lean:232`), and of existence of the large-field axial gauge, `Axial.existence` (`YM/Branch/Pick.lean:80`). The value of the threshold.

- Where the body of the explicit renormalisation map for the main terms lives. As a one-step statement, we could read only as far as `History.Transfer` (`YM/RG/History.lean:223`).
- The concrete formula for the side b_m.
- Whether a statement of reflection positivity for the one-step kernel exists anywhere in the repository (we have not searched all of it).

What this chapter can say

Can say	Cannot say
Source One step of the published construction is a probability kernel, and coarse links are drawn with the penalty $e^{-\beta\, c_{\mathrm{pen}}\ \mathrm{price}}$ around the predictor. The side is an odd number at least 3, and blocks are centred on coarse points	The content of the proofs of existence and uniqueness of gauge fixing
The Lean of this book's study: reflection positivity of the Wilson measure pushed forward by the side-3, centred, uniform map (`Block3.rp_push`). L even, m ≤ n steps, all real β, bounded measurable functions on the open positive side	The version for planes through link midpoints, infinite volume, the mass gap, varying the lattice spacing
The Lean of this book's study: a matched sequence exists for the side-3 design (`Block3.tuned3_exists`)	Monotonicity or divergence of the sequence of couplings, uniqueness of the solution
Source The published construction's statement of reflection positivity concerns the Wilson measure of the original lattice. A statement of reflection positivity for the one-step kernel is not within what we read	We have not searched the whole repository
Inferred Reflection positivity of the pushed-forward measure is not needed in the published construction's skeleton because what is taken to the limit are correlations under the measure of the original lattice	Whether the measure pushed forward with the published construction's design is reflection positive

Sources and reproduction

Item	Kind	Source / tool
One-step kernel, predictor, gauge fixing, symmetry	Source	In the repository `yang-mills-group/OpenAI-YM-20261028` (Apache-2.0): `YM/RG/Block.lean` (:71 `density`, :77 `push`), `YM/RG/Patch.lean` (:87 `weight`, :159 `bondCost`, :171 `usesBest`), `YM/Branch/Center.lean:72` (`min_iff`), `YM/Branch/Covariant.lean:50` (`channel`), `YM/Lattice/Subdivision.lean:29` (`embed`), `YM/Lattice/Scale.lean:28` (`Schedule`)

Item	Kind	Source / tool
Definition of the correlations and the limit	Assessment	The assessment (`docs/assessment-1.pdf` in `yang-mills-group/YM`, 2026-11-10), equation (6) and Theorem 4.1 in § 4.3
Reflection positivity of the original lattice	Source	`YM/Slice/Reflection.lean:98` (`reflected_positive`), `YM/Wilson/Law.lean:87` (`law`), `YM/Extraction/Harvest.lean:46` (`realPositive`)
This book's current design	Lean	Table of sources in 19-04
Theorems for the side-3 design	Lean	The Lean bundle of this book's study (`Block3.paths`, `wts`, `blockMap_refl_cfg`, `down_refl`, `rp_push`, `rp_push_prob`, `down_flipSet`, `Y3_zero`, `tendsto_Y3_one`, `tuned3_exists`). Passed the kernel recheck and checked in the source
Numerics for commutation of the side-3 map	Reading	Numerical record of this book's study (non-abelian, four-dimensional, sphere values, two steps: differences in commutation with reflection, exchange of axes and coarse translations $\leq 1.1 \cdot 10^{-14}$; control maps show breaking of 0.26–1.8)

Terms used in this chapter — renormalisation step, block map, push-forward measure, probability kernel, predictor, projection onto the group, penalty coefficient, gauge fixing in two strands, growing sides, reflection positivity, matching. Their meanings are in the A-02 glossary.

Next chapter: 14-03 Comparing matchings — we compare how the coupling is fixed after one step.

2026-11-13 · chapter Yang–Mills and the mass gap Part 14 Comparison — this book's Lean, and ideas this book's study lacked matching · Kiichi, Shiori and Rin

Comparing matchings — a quantity fixed by the measure, and a coefficient in the representation

> This chapter is this book's own investigation. The descriptions of the published construction (§ 03– § 05) are established knowledge (a reading of the published construction). § 06 is inference.

What this chapter shows — The type "choose the sequence of bare couplings so that a quantity determined at some coarse step takes a fixed value" is the same on both sides. What differs is the quantity matched (in this book's study, quantities determined by the measure alone, such as the expectation of the average of coarse plaquettes or the vortex ratio; in the published construction, the coefficient of F^2 in the representation of the density at step 0) and how existence is derived (the intermediate value theorem from global endpoints / shooting with nested bands at each step). In the published construction, this matching plays the role of an "anchor" in the uniqueness argument for convergence in the cutoff. From the presence or absence of this anchor, we infer why the limits of this book's study could be taken only along subsequences.

Prerequisite chapters — 19-07 § 02 matching, 10-01 what the coupling at step m is, 10-04 the shooting method, 28-02 implication of full-sequence convergence.

Said first — The matching theorems of this book's study state only the existence of a solution on each lattice at fixed lattice spacing; they say nothing about monotonicity, divergence or uniqueness of the sequence of couplings, or about a running law (the coupling growing in proportion to $\log(1/a)$). § 06 is inference, not a confirmed cause of the subsequence ambiguity. The agreement of one-loop coefficients is a hand computation with matched conventions.

The order of this chapter

1. Matching as a type — the coupling is not an input but a number for fixing the scale
2. This book's study's definition — a quantity determined by the measure alone — the expectation of the average of coarse plaquettes, the vortex ratio
3. The published construction's definition — a coefficient in the representation — two normalisations, and rereading

4. How existence is derived — global endpoints and the intermediate value theorem / shooting with nested bands
5. Its role as an anchor — uniqueness for a two-point boundary value problem, and convergence of the full sequence
6. Why this book's limits could be taken only along subsequences — inference
7. Comparing one-loop coefficients — they agree once conventions are matched
8. What is not said, and what we have not read

Labels: Source = we read the source of the published construction; Inferred = this book's inference; Assessment = from the committee's assessment. The other labels (Proved here, Cited, Reading, Lean and so on) mean the same as in the rest of the book (details in "Labels" in 6-00). For Lean theorems of this book's study, the name and scope are written on the same line, outside the labels.

01

Matching as a type

In the continuum limit, the lattice coupling β is not an input but a number adjusted to calibrate the lattice spacing (introduction in 5-05). If, on a lattice with spacing a_N, one chooses the bare coupling β_N so that "a quantity Y determined by a single length independent of the spacing a_N takes a fixed value κ_0", that length becomes the unit of scale. We call this matching.

The two sides share this type Inferred . They differ in three respects.

1. What the matched quantity Y is (§ 02, § 03).
2. How existence of a solution is derived (§ 04).
3. Where the matched coupling is used in the argument for the limit (§ 05).

02

This book's study's definition — a quantity determined by the measure alone

This book's study defines the matched quantity from the measure alone (19-07 § 02). Two examples are in Lean.

The expectation of the average of coarse plaquettes. Push the Wilson measure μ^β on the fine lattice at step $m_0 + k$ forward to step m_0 by the composition of k steps of the

block map, and take the expectation of the average real part Y_0 of coarse plaquettes.

$$Y(m_0, k, \beta) = \int Y_0 \, \mathrm{d}\big(\mathrm{down}_* \mu^{\beta}\big) \quad (1)$$

In the side-3 design (14-02 § 03), $Y(m_0, k, 0) = 0$ (`Block3.Y3_zero`), $Y \to 1$ as $\beta \to \infty$ when $0 < r_0 \le 1$ (`Block3.tendsto_Y3_one`), and under $0 \le \kappa_0 < 1$ and $3^{m_0} L \ge 2$, for every k there is $\beta_k \ge 0$ with $Y(m_0, k, \beta_k) = \kappa_0$ (`Block3.tuned3_exists`).

The vortex ratio. $\theta_M = (Z_0 - Z_1)/(Z_0 + Z_1)$, built from the twisted partition functions Z_0, Z_1 of a torus of width N and height M (19-02), is a quantity that does not use the block map. θ_M is continuous in β with $\theta_M(0) = 0$ (`ThetaTuning.thetaM_continuous`, `ThetaTuning.thetaM_zero`), tends to 1 as $\beta \to \infty$ with the lattice fixed (`ThetaLimit.tendsto_theta_one`), and for every $\kappa_0 \in [0, 1)$ there is $\beta \ge 0$ with $\theta_M(\beta) = \kappa_0$, the set of solutions having a largest element (`ThetaLimit.tuning_all`). Details are in 21-07.

The advantage of this definition is that it can be written without first fixing a representation or an approximation. In exchange, the relation between "the value of the observable and the coefficient of F^2 in the effective action" has to be shown separately.

03 The published construction's definition — a coefficient in the representation

In the published construction, the effective density at step m of the theory with cutoff N and volume p is written in a fixed form of representation

$$\mathrm{eval}_m = \exp\big(v|\Lambda| - \beta K - B - A\big) \cdot \mathrm{gas} \quad (2)$$

(K the main action, B the one-loop part independent of β, A a local remainder, gas the gas of large-field activities), and the coupling at step m is defined as the coefficient β there Source (details in 10-01). So that the F^2 term cannot be moved between βK and B, two normalisations are imposed: the zero-momentum coefficient matrix of the quadratic part of K is the identity, and the zeroth- and first-order Fourier coefficients of B are 0 Source .

The docstring in the source describes this β as "an actual number in the representation, not a number tied to the measure independently" (top of `YM/RG/History.lean`) Source . The number itself is reread from the log-ratio of the density at the flat configuration 1 and at a test configuration V,

$$\log H(1) - \log H(V) = \beta K(V) + B(V) + A(V) \quad (3)$$

(`contrast` in `YM/UVNext/COUPLING_RESET/Reset.lean:50`) Source . The right-hand side increases in β with slope $K(V) > 0$, so the root is unique and depends continuously on the input Source (10-03).

As for the direction of steps, $m = N$ is the finest, where the representation is the bare Wilson weight itself, and $m = 0$ has spacing 1 Source . The matching is "the coupling at step 0 equals a fixed reference value $\bar{\beta}$" Assessment (the reference conventions in Assessment § 11 and Appendix A). The reference value is taken large, keeping weak coupling $\beta_m \in [\hat{\beta}_m/2,\ 2\hat{\beta}_m]$ at every step (field `band` of `History.Conv`) Source .

	This book's study	The published construction
Matched quantity	Expectation of the average of coarse plaquettes under the push-forward measure; the vortex ratio	The coefficient of F^2 in the representation of the density at step 0
What the definition needs	The measure and the block map (the vortex ratio does not even need the map)	A representation preserved at every step, and two normalisations
Where it is read	An integral over all configurations of the coarse lattice	A log-ratio at a small-field test configuration
What must be shown separately	The relation between the value of the observable and the coefficient of F^2	That the representation is preserved at every step (induction)

04

How existence is derived

This book's study: global endpoints. If the matched quantity $Y(\beta)$ is continuous in β and its values at $\beta = 0$ and as $\beta \to \infty$ are known, the intermediate value theorem gives a solution for any target between them. In the side-3 design the value at $\beta = 0$ is 0 by symmetry, and the value as $\beta \to \infty$ is 1 because flat configurations are pushed forward to flat ones. Only the values of one continuous function at its two ends are used.

The published construction: nested bands at each step. Take the bare coupling as a parameter x, and view the coupling at step m as a continuous function $f_m(x)$ of x. On the existence path (the path whose chain to the main theorem we checked), one uses the band at each step

$$\mathrm{low}_m = \tfrac{3}{4}\,\bar{\beta} + \tfrac{5}{8}\,s\,\log D_m, \qquad \mathrm{high}_m = \tfrac{5}{4}\,\bar{\beta} + \tfrac{3}{2}\,s\,\log D_m \quad (4)$$

(s the one-loop slope, D_m the product of the sides up to step m)
(`YM/FullVolume/Arc.lean:21`, :22) Source . From the one-step error inequality, inside an interval that reaches both ends of the band at step $m+1$ one can take a subinterval that reaches both ends of the band at step m too, and nested intervals are obtained

going downward from step N (`Arc.bought`, same file :95) Source . At step 0, $\bar{\beta}$ lies inside the interval $[\frac{3}{4}\bar{\beta}, \frac{5}{4}\bar{\beta}]$, so the intermediate value theorem (`covers`, same file :71) gives a bare coupling for which the coupling at step 0 is $\bar{\beta}$ Source . On the existence path, continuity of f_m is supplied by `Roof.Lyrics.supplier` (`YM/FullVolume/C/Songbook.lean:76`) Source .

The repository also has shooting with a different band $[\bar{\beta} + \frac{s}{2}\log D_m,\ \bar{\beta} + 2s\log D_m]$ (under `YM/Shoot/`), but the path to the main theorem (`rodB` in `FullVolume/Stage/Rods.lean:26`) does not use it Source . The detailed form of the shooting method is in 10-04.

To summarise the difference: this book's study uses the global endpoints of one function, while the published construction uses local bands at each step and the one-step error inequality. The published construction's method works in the same form regardless of the number of steps, given the condition that the sum of one-step errors stays within a small fraction of the band width (the calendar condition). This book's method can be used only for quantities whose endpoint values are known from symmetry or limits Inferred .

05

Its role as an anchor

In the published construction, matching does more than "fix the unit": it enters the argument that the ultraviolet limit at fixed volume can be taken along the full sequence Source .

The source of convergence in the cutoff is not a quantitative estimate comparing N and N', but three things: compactness, uniqueness of the limit, and the subsequence principle (`Window.allB`, `YM/FullVolume/Glide/Triumph.lean:37`) Source . The sequence of ingredients of the representation (couplings and activities) lies in a compact set, so it has subsequential limits. Every subsequential limit satisfies the "ladder equations" across all steps $m \in [0, \infty)$. If these have only one solution, all subsequential limits coincide and the full sequence converges.

Uniqueness (`accord`, `YM/FullVolume/Glide/Coincide.lean:58`) has the form of a two-point boundary value problem Source .

- **The marginal direction (the coupling)** is anchored at step 0. By shooting, the coupling at step 0 is $\bar{\beta}$ for every N, p, so the difference between the couplings of two solutions is 0 at step 0, and propagates towards the fine side through the sum of one-step deviations. The size of that sum is bounded by the calendar condition to a small fraction of the band width Source .

- **The irrelevant directions (the activities)** shrink by $1/16$ per step from the fine side to the coarse side (`Window.curls`, `YM/FullVolume/Glide/Duty.lean:45`; `Muse.ticket`, `YM/FullVolume/Gain/Rate.lean:94`) Source .

If b is the supremum of the difference between two solutions, these two facts give $b \leq b/16$, so $b = 0$ Source . Differences in the initial values at the fine end are forgotten by $(1/16)^k$ as one goes down the steps, so they need not be small, only bounded (`culprit`, `YM/FullVolume/Glide/Wings.lean:59`) Source . Estimates of this amount of forgetting are also used for convergence of the activities Source . There is no rate of convergence in the statement (qualitative convergence) Source .

Without the anchor, the coupling direction does not contract, so this argument does not close. The matching that sets the coupling at step 0 to the same $\bar{\beta}$ for every cutoff is exactly that anchor Inferred . Details are in 11-10.

06

Why this book's limits could be taken only along subsequences

This section is inference Inferred .

In the implication of subsequential limits of this book's study (28-01), the first conclusion states that "the matching point lies in the bracket" (a point where $u_n(x) = t$ for the family of functions u_n), but there was not a single field linking u_n to the fine measure, so the sequence of bare couplings and the matching were not tied together. The implication of full-sequence convergence (28-02) had in Lean, for orbits of an abstract skew-product map (marginal coordinates and contracting irrelevant coordinates), the form "if the marginal coordinate at the coarse end is matched to the same value for every n, and the irrelevant coordinates contract per step, then the coarse end is Cauchy", and also showed by example that the matching at the endpoint cannot be removed. This is the same type as the two-point boundary value problem of § 05. However, that map was not defined from the lattice.

It is therefore possible that the limits of this book's study could be taken only along subsequences because there was no anchor in the coupling direction (the matching lay outside the chain). The existence of a matched sequence for the side-3 design (§ 02) is a concrete object that could be a candidate for an anchor. However, it has not been shown that it is a matching of the marginal coordinate of a one-step map. There are also two differences from 28-02. The marginal coordinate in the 28-02 theorem is not set equal to either u_n of 28-01 or the matched quantity Y of the side-3 design. And convergence in 28-02 is a quantitative Cauchy property (with a rate) obtained from Lipschitz contraction, derived differently from the published construction's "compactness and uniqueness" (without a rate).

The anchor alone is not enough, however. The uniqueness argument of the published construction rests on a representation preserved at every step (the ingredients splitting into couplings and activities) and on the per-step contraction of the activities. It has not been shown that, with this book's deterministic block map, the coarse density keeps a representation of this form at every step. This is where it overlaps with the treatment of large fields (14-04).

07

Comparing one-loop coefficients

The slope of the one-step shift of the coupling in the published construction is the constant set by definition

$$s = \frac{11\, C_g}{24\pi^2} \quad (5)$$

(`slopeConstant` in `YM/Gauge/Colors.lean:184`) Source . The assessment states that the bare coupling satisfies $\beta_{N,p}/\log(1/aN) \to 11C_g/(24\pi^2)$ uniformly in the volume Assessment (equation (7) in Assessment § 4.3). What the published statement `WilsonRegularity.running` (`ChallengeSpec.lean:13`) requires is only "there is some positive slope"; $11C_g/(24\pi^2)$ is the value that the construction satisfies Source . C_g is the Casimir constant for the inner product $\langle X, Y\rangle = -\mathrm{Re}\,\mathrm{Tr}(XY)/d$ (d the dimension of the representation) Source .

For the two-dimensional representation of SU(2), $C_g = 8$ and $s = 11/(3\pi^2)$, and the shift for one step with side 2 is $s \ln 2 = 11 \ln 2/(3\pi^2) \approx 0.2575$ Proved here (hand computation; $C_g = 8$ also checked numerically). This agrees with the one-loop value that this book's study wrote in units of β (15-01) Proved here . The premise is that this book's β is the coefficient of $\sum_p (1 - \frac{1}{2}\mathrm{Re}\,\mathrm{tr}\, U_p)$ (the standard $\beta = 4/g^2$). The published construction uses only odd sides of at least 3, so it does not treat the side-2 step itself. Only the coefficient of $\log b$ agrees; the $O(1)$ constants of one step cannot be compared.

The published construction leaves the one-step error at $O(1)$, lets the side b_m grow with the step and removes it by dividing by $\log b_m$ (10-05) Source . This book's study kept the side fixed, so it had a separate problem: the $O(1)$ error of one step accumulating over the number of steps (the deviation of the coefficient at the first step measured in 15-01).

08

What is not said, and what we have not read

What this book's study does not say. Uniqueness of the matching solution, monotonicity or divergence of the sequence of couplings, a running law, estimates on θ_M uniform in N. The matching of the vortex ratio is a statement with the lattice fixed, and says nothing about sequences in which the lattice and the coupling move together.

Where the short-distance effective coupling lives. For Theorem 7.2 of the assessment (the short-distance effective coupling $\log(1/r)\, g_W(r)^2 \to 1/(2b_0)$; in the assessment's notation, $2b_0$ corresponds to the slope s in (5)), there are three declarations with similar content Source . The assessment does not name a location Assessment .

Declaration	Content	Dependency
`YM.PertExtension.Running.ratio` (`YM/PertExtension/Running.lean:145`)	Ratio form ($\mu \to \infty$, $a_m \le 1/\mu$)	Outside the dependencies (outside) Source
`YM.Trace.Pilot.wilson_link` (`YM/TraceExtension/Atlas/Sink.lean:50`)	Wording close to the statement of Theorem 7.2 ($r \downarrow 0$, the first spacing below r) Inferred . `PostT` ($b_i \le \log \hat\beta_i$) among the hypotheses	Inside the dependencies. First headline `continuum_with_position_tests`, direct user `crest_ready`. Used as an intermediate step in the trace analysis; its conclusion enters `Crest.charts` and `FieldComparison.wilson` and is dropped by `of_result` Source
`YM.Pert.Asymptotics.comparison` (`YM/Pert/Asymptotics/RawAxis.lean:36`)	The running of the coupling of the cut-off step i, $\beta(N_i, k_i, i)$ $-1/2$; a different quantity from g_W	A headline theorem itself (headline) Source

A statement in the form of g_W does not survive into any conclusion of the published theorem, the construction's theorem or the committee's joint goal Source . The headline theorem `comparison`, too, is one of the 17 headlines listed in the verification records and is not in the conclusions of the three goals (it is not referenced from the construction's theorem, the published theorem or the committee's joint goal) Source . Universality with respect to the choice of sequence of spacings, and the dependence on changing $\bar\beta$, are not stated Assessment Source .

What we have not read. The proof that connects the value of the one-loop shift with the kinetic term of the one-step theorem (beyond `under_contract`), the concrete form of the test configuration V, and how `Roof.Lyrics.supplier` produces continuity.

What this chapter can say

Can say	Cannot say
The Lean of this book's study: the vortex ratio θ_M is continuous in β with $\theta_M(0)=0$, tends to 1 as $\beta\to\infty$, and for every $\kappa_0\in[0,1)$ there is a largest solution of $\theta_M(\beta)=\kappa_0$ (`ThetaTuning.thetaM_continuous`, `thetaM_zero`, `ThetaLimit.tendsto_theta_one`, `tuning_all`; lattice fixed)	Uniqueness of the solution, continuity in κ_0, estimates uniform in N
The Lean of this book's study: in the side-3 design there is a sequence matching the expectation of the average of coarse plaquettes (`Block3.tuned3_exists`)	Monotonicity, divergence or boundedness of the sequence; a running law
Source The docstring in the source says that the step coupling of the published construction is a coefficient in the representation of the density, not a number determined by the measure alone	The dependence when the representation is changed
Source Shooting on the existence path uses nested intervals of the band $\frac{3}{4}\bar\beta+\frac{5}{8}s\log D_m$ to $\frac{5}{4}\bar\beta+\frac{3}{2}s\log D_m$, and takes $\beta=\bar\beta$ at step 0	Uniqueness of the bare coupling chosen by shooting
Source Convergence in the cutoff follows from compactness, uniqueness and the subsequence principle. Uniqueness comes from the anchor at step 0 and the contraction of the activities by 1/16 per step	The rate of convergence (not in the statement)
Inferred The subsequences in this book's study may come from the absence of an anchor in the coupling direction	Not checked. Even with an anchor, the same argument does not stand without the representation and the contraction
Proved here With conventions matched, the one-loop coefficient agrees at $\frac{11}{3}\frac{\ln 2}{\pi^2}$ for one step with side 2 (hand computation; $C_g=8$ also checked numerically)	Comparison of the $O(1)$ constants of one step

Sources and reproduction

Item	Kind	Source / tool
Docstring on the definition of the step coupling	Source	In the repository `yang-mills-group/OpenAI-YM-20261028` (Apache-2.0): the docstring at the top of `YM/RG/History.lean` (around :24), `Conv` (:213), `Transfer` (:223), `Selected` (:234)
Rereading (log-ratio of two configurations)	Source	`YM/UVNext/COUPLING_RESET/Reset.lean:50` (`contrast`)
Shooting method (existence path)	Source	`YM/FullVolume/Arc.lean` (:21 `low`, :22 `high`, :71 `covers`, :95 `bought`), `YM/FullVolume/C/Songbook.lean:76` (`Roof.Lyrics.supplier`), `YM/FullVolume/Stage/Rods.lean:26` (`rodB`)

Item	Kind	Source / tool
Anchor and contraction	Source	`YM/FullVolume/Glide/Triumph.lean:37` (`allB`), `YM/FullVolume/Glide/Coincide.lean:58` (`accord`), `YM/FullVolume/Glide/Duty.lean:45` (`curls`), `YM/FullVolume/Gain/Rate.lean:94` (`Muse.ticket`), `YM/FullVolume/Glide/Wings.lean:59` (`culprit`)
One-loop slope	Source	`YM/Gauge/Colors.lean:184` (`slopeConstant`). Equation (7) in Assessment § 4.3
Short-distance effective coupling (three locations with similar content)	Source	`YM/PertExtension/Running.lean:145` (`ratio`, outside), `YM/TraceExtension/Atlas/Sink.lean:50` (`wilson_link`, used), `YM/Pert/Asymptotics/RawAxis.lean:36` (`comparison`, headline). `verification/declaration-usage.tsv`, `verification/joint-headlines.txt`; on the assessment side, `submissions/openai-20261028/checks/CombinedAcceptance.lean`
Matching, bare coupling, running	Assessment	The assessment (`docs/assessment-1.pdf` in `yang-mills-group/YM`, 2026-11-10), § 4.3, § 7.4
This book's matching theorems	Lean	The Lean bundle of this book's study (`ThetaTuning.*`, `ThetaLimit.*`, `Block3.*`). Passed the kernel recheck and checked in the source
Comparison of one-loop coefficients	Proved here	Hand computation of this book's study (the value of C_g also checked numerically). The value on this book's side is in 15-01

Terms used in this chapter — matching, bare coupling, expectation of the average of coarse plaquettes, vortex ratio, coefficient in the representation, rereading, shooting method, band, anchor, two-point boundary value problem, forgetting of the upper end, running law, one-loop coefficient. Their meanings are in the A-02 glossary.

Next chapter: 14-04 Large fields — we compare the treatment of large fields, which was open in this book's study, with that of the published construction.

2026-11-13 · chapter Yang–Mills and the mass gap Part 14 Comparison — this book's Lean, and ideas this book's study lacked large fields · Kiichi, Shiori and Rin

Large fields — where this book's study got stuck, and how the published construction handles them

> This chapter is this book's own investigation. Descriptions of the published construction are established knowledge (a reading of the published construction), and § 01 is a map of open questions. The judgements in § 06 and § 07 are inference.

What this chapter shows — On the ultraviolet side of this book's study, large fields were what remained without a known way forward. In a form that stacks up the large-field regions step by step and fixes them, the treatment of defects that persist across steps near configurations whose shape the block map does not change was left open. The published construction keeps no history of regions and re-counts seeds at each step, clips the main part of the representation to linear growth, sets up at every step an "energy per seed" inequality for every configuration satisfying the gauge section and cell conditions, puts a weak ledger in the induction hypothesis and strong bounds in the one-step certificate, and discharges the step-uniform conditions on the calendar side. We read this as "answering in a different form" the point where this book got stuck.

Prerequisite chapters — 11-00 the induction hypothesis and the one-step certificate, 9-07 the small-field threshold and the clipped action, 28-04 the list of premises (large-field estimate, large-field gas), 22-01 rarity of large plaquettes.

Said first — For many of the published construction's large-field lemmas, tracing them through the record of how declarations are used leads to a headline theorem other than the main theorem. We have not checked whether they are among the dependencies of the main theorem (this does not mean that they are not). We have not read the whole construction that re-files the coarse-step activities into the ledger after one step, the lemma that pays for the Haar loss, or the bound on the remaining integral. The judgement in § 06 is an application to the vocabulary of this book's study; it does not mean that the proofs of the published construction were written with this book's questions in mind.

The order of this chapter

1. Where this book's study got stuck — regions stacked up and fixed, and persistent defects
2. The published construction's representation at each step — the clipped action, and a gas of activities only on flags
3. Inside one step — seeds and cells — seed types, cells, partitions of unity
4. Energy per seed — an inequality for configurations satisfying the section and cell conditions
5. Two ledgers and the calendar — a weak ledger and strong bounds, the stamp, step-uniform conditions
6. Judgement — answered in a different form — what can and cannot be said (Table 1)
7. Configurations that keep exceeding the threshold — how they are handled across steps
8. Clusters that return to small fields — the test of whether they contain a coarse flag
9. What we have not read, and checking dependencies

Labels: Source = we read the source of the published construction; Inferred = this book's inference; Assessment = from the committee's assessment. The other labels (Proved here, Cited, Reading, Lean and so on) mean the same as in the rest of the book (details in "Labels" in 6-00). For Lean theorems of this book's study, the name and scope are written on the same line, outside the labels.

01

Where this book's study got stuck

When a lattice field is coarsened step by step, at any step there may remain places where the field deviates strongly (where the holonomy of a plaquette is far from the identity). There the small-field expansion (Gaussian integrals and the expansion of their corrections) cannot be used, so a different treatment is needed. This is the large-field problem (7-05, 15-01).

Following the pattern of Bałaban's series, this book's study thought in terms of defining the large-field regions at each step and stacking them up, fixing them for later steps. In this form, defects that persist across steps may remain near configurations whose shape the block map does not change (configurations mapped to the same shape at

every step). For such defects, how to show that the gain in action and the loss from fluctuations do not balance out, so that the rarity at each step acts uniformly in the number of steps — this remained unknown.

In the list of premises (28-04), this part was placed in two rows, the "large-field estimate" (at each step and each hole, activities are bounded by the exponential of the lower bound of the action, with constants uniform in the step and the volume) and the "large-field gas" (an estimate of the gas of large-field activities, and its equality with the corresponding ratio), and neither had a settled form Assumption (28-06). Rarity at one scale (under the fine-lattice Wilson measure, the probability of the set of large plaquettes is small uniformly in the volume) is a Lean theorem (`FoldCap.rarity_explicit`, 22-01), but it does not reach a form that spans steps.

02

The published construction's representation at each step

The published construction keeps the effective density at each step m in the following form Source (details in 9-06, 11-01).

$$\mathrm{eval}_m = \exp\Big(v|\Lambda| - S_m^{\mathrm{clip}} - \sum_i 1[\mathrm{healthy}]\, E_i\Big) \cdot \mathrm{gas}_m(K) \quad (1)$$

S^{clip} is the clipped action, E_i the small-field (regular) activities, and $\mathrm{gas}(K)$ the gas of large-field (forced) activities K.

Threshold. From the reference coupling $\hat{\beta}_m$ at step m, set $\hat{g}_m = \hat{\beta}_m^{-1/2}$ and $p_m = \exp\big((\log\log\hat{\beta}_m)^2\big)$, and let the threshold be $t_m = \hat{g}_m\, p_m$ (`YM/RG/History.lean:81`, :83) Source. For sufficiently large $\hat{\beta}$ (in the asymptotic sense), p grows faster than any power of $\log\hat{\beta}$ and slower than any positive power of $\hat{\beta}$ Proved here. For example, at $\hat{\beta} = 10^{10}$, $p \approx 1.9 \cdot 10^4$, which is larger than $\hat{\beta}^{0.1} = 10$, so the slower growth becomes visible only for much larger $\hat{\beta}$ Reading.

Clipped action. For each plaquette, in terms of its deviation s (the distance of the plaquette holonomy from the identity),

$$\mathrm{clipped}(s) = \begin{cases} \dfrac{c_{\mathrm{dir}}}{2}\, s^2 & (s \le t) \\ c_{\mathrm{lin}}\, t\, s & (s > t) \end{cases}, \qquad c_{\mathrm{lin}} < \frac{c_{\mathrm{dir}}}{16} \quad (2)$$

(`clipped` at `YM/RG/PathActions.lean:218`, `kink` at :145) Source. It is quadratic up to the threshold and linear beyond it, jumping down at $s = t$.

Activities only on flags. The large-field activities K live only on clusters that exactly cover the places exceeding the threshold at the current step (the flags), and if there are

no flags the gas is 1 (`gas_one` in `YM/RG/Activities.lean:135`) Source . What the induction hypothesis `History.Conv` (`YM/RG/History.lean:213`) holds is the ledger of small-field activities $\hat{g}^{11/5}$ (:84), the ledger of large-field activities

$$\sum(\text{cover-weighted } K) \;\le\; \exp\!\left(-p_m^{1/5}\right) \quad (3)$$

(`forcedAllowance` at :85), and the coupling band Source . **There is no field representing a history of regions** Source .

Between steps, an identity. The densities at steps m and $m+1$ are linked by the push-forward identity with the one-step kernel, $\mathrm{eval}_m = \mathrm{push}(\mathrm{eval}_{m+1})$ (`History.Transfer`, :223) Source . Clipping the main part of the representation does not change the measure; the clipped difference goes into the small-field and large-field activities Source .

At the finest step, the Wilson price of each plaquette is split into the clipped action, the quadratic part on healthy places, the remainder on healthy places, and a balance (due), and the balance is charged to the first large-field activities (`due_eq` in `YM/Extra/WILSON_STARTUP/Ownership.lean:68`) Source . On flagged plaquettes the balance is at least $t^2/4$ Source (11-02).

03 Inside one step — seeds and cells

Inside the integral of one step, configurations are not cut into "small fields and large fields" by characteristic functions Source . Three kinds of "seed" are counted: fine plaquettes exceeding the threshold, coarse links deviating strongly from the predictor, and points touching a coarse flag Source . A cell is built for each seed type (a triple of finite sets), and the cells are glued together by a smooth partition of unity Source (11-03).

The cell condition itself (`Seeded`, `YM/Push/SECTOR_MEASURE/Cells.lean:51`) is imposed on all plaquettes and links Source . What the estimates use is a form localised around the seeds, `Plot` (`YM/Pressure/Seeds.lean:51`), with localisation radius $5\lceil(\log\hat{\beta})^2\rceil$ in terms of the reference coupling $\hat{\beta}$ on the coarse side Source . The boundaries of cells are written with two values, $t/4$ and $t/2$, for the plaquette condition, giving a band of offset (hysteresis) Source .

04 Energy per seed

The key inequality of one step is, on a cell,

$$c_E\, t_{\mathrm{f}}^2 \cdot \#(\text{seeds}) \;\le\; \big(\text{fine action} + \text{deviation penalty} - \text{coarse clipped action}\big)_{\text{around seeds}} \quad (4)$$

(`cash`, `YM/Pressure/Seeds.lean:124`; `Grants`, same file :129) Source . t_{f} is the threshold on the fine side Source . The order of quantifiers is "there is i such that for every side L at least i and every sufficiently large W there is $c_E > 0$ such that for every sufficiently large reference coupling, ...", and c_E depends on L and W Source (κ in the source's wording). The statement is made, with patch side L, lattice side c at least $100(1 + \lceil(\log \hat{\beta})^2\rceil)$, and the hypothesis `Funds` on the choice of kernel and so on, for **every configuration** satisfying the section with the gauge fixed on a tree (`SliceSet`, :48: $U = 1$ on the trunks of the forest) and the cell condition `Plot` (`live`, :113–:118) Source . This structure holds for every choice without hypotheses (`grants`, `YM/Next/FLUCTUATIONS/Grants.lean:24`) Source .

Multiplying by β, since $\beta t^2 \approx p^2$, each seed carries a factor of about $\exp(-c_E p^2)$ Inferred . Since p^2 grows faster than any power of $\log \beta$, polylogarithmic losses such as the volume around a seed, the Haar loss and combinatorial counts can be paid with this factor Inferred .

One of the positive remainders in inequality (4) comes from the clipped action. For coarse plaquettes of intermediate size (larger than the coarse threshold t_{c} and at most some δ), a positive term $\frac{c_{\mathrm{dir}}}{4}\, t_{\mathrm{c}} \cdot (\text{deviation})$ appears in the lower bound for the work on the coarse side (`Crowds` at `YM/Bid/Energy.lean:94`–:97, `E05` at :101) Source . The shape is that, since the action handed to the coarse step is clipped to linear growth, a positive difference from the quadratic action on the fine side remains Inferred .

05 Two ledgers and the calendar

A weak ledger and strong bounds. The large-field ledger held by the induction hypothesis `Conv` is only the weak form (3) Source . The one-step certificate `Move` (`YM/PCalc/Band.lean:49`) states, for the exit activities, the much stronger

$$K_{\text{exit}} \;\lesssim\; \exp\!\big(-g \cdot p_{m+1}^2\big) \quad (5)$$

(field `forced`; p_{m+1} is the value at the fine step) Source . Dropping the strong bound to the weak ledger is a short inequality: for all sufficiently large y, if x and y are "alike" (six quantities — threshold, P, gap, number of clusters, ledger w and so on — are within a factor of 2 of each other; `Alike`, `YM/Next/SCHEDULE/Dues.lean:118`), then $\exp(-d\,P(y)^2) \le w(x) = \exp(-P(x)^{1/5})$ (`saved`, `YM/Next/SCHEDULE/Salary.lean:178`; `late_force`, `YM/AnalyticBridge/VERIFIED_RESET_STEP/Opening.lean:19`) Source . The proof of `saved` handles the exponent $1/5 \le 1$ (using that the ratio of the P values is

within 2), and the form stating the same for any exponent below 2 is `dictionary_fit` (`YM/AnalyticNext/CONNECTED_ACTIVITY_JETS_E6/Bulkhead.lean:78`) Source . The strong estimate is not put into the induction hypothesis; it is the weak ledger that is rebuilt at every step. The step-uniform conditions are discharged on the calendar side, as in "Calendar" below Inferred .

The stamp. Of the income from one seed, half of the guaranteed lower bound $c_E P^2/2$, namely $c_E P^2/4$, is left at the exit as a "stamp", and the rest pays for the losses of the tree expansion and the combinatorics of covers (`Target.costs`, `YM/AnalyticNext/CONNECTED_ACTIVITY_JETS_E6/OptionCost.lean:83`, $u = 1/2$) Source (11-05).

Incoming activities. The formula for the exit rate (the stamp and the target rate) is a function of c_E, p, β only, and estimates of the incoming activities do not enter it Source . Estimates of the incoming activities are used on the side of the one-step hypotheses (a field of `Target.Invoices`, `YM/AnalyticNext/CONNECTED_ACTIVITY_JETS_E6/Issue.lean:73`) to make the sums inside one step converge Source . Rarity is rebuilt at every step from the action difference of that step Inferred .

Calendar. The weak ledger is also a function of the step m. The step-uniform conditions are discharged on the calendar side. The lower bound on sides, the entry point of the reference coupling, the allocation of rates and so on are fixed in advance, in the order "step constants → sides → widths → ... → entry point of the coupling" Source (11-08). On the existence path, carrying the ledger from step to step is done by `Roof.Lyrics.supplier` (`YM/FullVolume/C/Songbook.lean:76`); the similarly named `Directed.holding` (`YM/PCalc/Band.lean:92`) is not used Source .

06

Judgement — answered in a different form

We read the published construction as **answering in a different form** the point where this book got stuck (§ 01) Inferred .

Table 1 Where this book's study got stuck, and how the published construction handles it

Point	This book's study	The published construction
Large-field regions	Defined at each step, stacked up and fixed thereafter	No history; at each step seeds are re-counted from that step's flags Source

Point	This book's study	The published construction
How to split	Small-field characteristic functions	Cells by seed type, and a smooth partition of unity Source
Main part of the representation	Quadratic action as the main part	Action clipped to linear beyond the threshold; the difference goes to the activities Source
Threshold	A polylogarithmic multiple of the coupling	$\hat{g} \cdot \exp((\log\log\hat{\beta})^2)$ Source
Factor per seed	Of type e^{-c/g^2}	Of type $\exp(-cEp^2)$. Much weaker, but the ledger $\exp(-p^{1/5})$ is weaker still Source Inferred
Persistent defects	How to bound them uniformly in the number of steps was open	An inequality for every configuration satisfying the section and cell conditions is set up at every step, producing the factor at every step Source Inferred
Step-uniform conditions	Placed in the premise fields Assumption	Discharged on the calendar side (lower bound on sides, entry point of the coupling and so on) Source
Instantons, vortices, topological sectors	Listed as candidates for persistent defects	No argument treating them by name was found on the path to the main theorem Source (within the range searched)

What can be said. The published construction keeps no history of regions, re-counts seeds at each step, and produces the factor per seed at every step through inequality (4), stated for every configuration satisfying the gauge section and cell conditions. Clipping the main part of the representation to linear growth is consistent with the positive remainder of the inequality (intermediate coarse plaquettes).

What cannot be said. One cannot say that "the premise of the configurations this book feared no longer applies". What is clipped is only the main part of the representation; steps are linked by identities, and the measure does not change. The clipped difference goes to the activities. Nor can one say that "the problem of estimates uniform in the number of steps does not arise". The weak ledger is also a function of the step, and the step-uniform conditions are discharged on the calendar side. What can be written is only that it answers in the form "clip the main part of the representation, put the difference into the activities, set up an inequality for every configuration satisfying the section and cell conditions, and discharge the step-uniform conditions with the calendar".

07

Configurations that keep exceeding the threshold

How does the published construction handle configurations of the type this book feared, for example a configuration that just exceeds the threshold of every step at every step?

The energy-per-seed inequality (4) is stated for every configuration satisfying the section and cell conditions, with no hypothesis excluding types of configuration Source . For such a configuration too, at each step the fine flags become seeds, and since there is at least one seed, the right-hand side of (4) is at least $c_E t^2$ Source (quantifiers) Inferred (application). At each step a stamp is attached, and the exit activities fit the strong form (5) and then the weak ledger (3) Inferred .

The rarity of continuing over many steps is not multiplied across steps. The rarity paid at a previous step is dropped to the weak ledger, and new rarity is extracted from the action difference at each step Inferred . Since the ledger has the same form at every step, the induction runs on that. The step-uniform conditions do not disappear; they are discharged on the calendar side (§ 05). On the action side, the coarse action term used at step m is the same as the fine action term at step $m-1$, so each step only extracts the action difference of that step and nothing is counted twice — this is how we read it Inferred . We have not checked the exact form including the symmetric averaging.

08

Clusters that return to small fields

After one step, a defect may return to small fields at the coarse step. The published construction separates clusters that contain a coarse flag (`ThickP`) from those that do not (`ThinP`) by a single local test (`YM/AnalyticNext/CONNECTED_ACTIVITY_JETS_E6/Colony.lean:26` , :27) Source . Only clusters containing a coarse flag become large-field activities K at the next step; the contribution of clusters without one goes into the terms of the small-field expansion (Ursell terms), whose value at flat configurations flows into the vacuum term v and whose quadratic part flows into the correction of the coupling Source (classification and the formula for the vacuum term) Inferred (the flow) (11-06).

In this book's study, "what to do if a defect returns to small fields" was a question of case analysis; in the published construction it is settled by one test inside the procedure that rebuilds the representation at the coarse step Inferred .

09

What we have not read, and checking dependencies

- **Dependencies.** What we confirmed by chain to lie on the path of the main theorem (`jointExistence`) is limited to the path of shooting and the one-step theorem (from `rodB` to the rereading), convergence in the cutoff (`allB` and others), `sailsW` , which closes the small-field conditions, and the ultraviolet limit `realUV` . For many of the large-field lemmas cited in this chapter (`late_force` , `due_eq` and others),

tracing them through the record of how declarations are used leads to a headline theorem other than the main theorem. This does not mean they are not on the main theorem's path; it means we have not checked.

- The whole construction that rebuilds the coarse-step activities after one step and files them into the ledger (3) (under `YM/PhysicalNext/RG_NEXT_COMPLETE_REPEATABLE_NONLINEAR_STEP/`, `V9_JOINT_ENDPOINT_LOAD`, `ANNULUS_RATIO_INSERTIONS`).
- Where the lemma lives that pays for the loss where integration stays with Haar. We read it as entering the field for the logarithmic cost per seed, but have not identified the concrete lemma.
- The whole bound on the remaining integral after the prices are extracted.
- The proofs of inequality (4) and of the positive remainder (we read only the statements).
- The map by which the values of clusters without a coarse flag enter the slots of the small-field activity ledger.

What this chapter can say

Can say	Cannot say
Source The induction hypothesis `History.Conv` has no field for a history of regions, and large-field activities live only on the flags of the current step	The whole construction that re-files activities into the ledger after one step
Source The main part of the representation is quadratic when the plaquette deviation is at most the threshold and linear beyond it, with the linear coefficient smaller than 1/16 of the quadratic one. Steps are linked by push-forward identities, and the measure does not change	All of how the clipped difference is distributed to the activities
Source The energy-per-seed inequality is stated for every configuration satisfying the gauge section and cell conditions (with hypotheses `Funds`, `SliceSet`, `Plot`)	The content of its proof (we read only the statement)
Source The induction hypothesis holds only the weak ledger $\exp(-p^{1/5})$, and the one-step certificate produces the strong bound $\exp(-g\,p^2)$. It is weakened by a one-line inequality	The whole part that rebuilds the gas of activities from the strong bound
Source Estimates of the incoming activities do not enter the formula for the exit rate. They are used on the side of the one-step hypotheses	—
Inferred The published construction "answers in a different form" the point where this book got stuck	One cannot say "the configurations this book feared do not occur" or "the premise no longer applies"

Sources and reproduction

Item	Kind	Source / tool
Threshold, ledgers, induction hypothesis	Source	In the repository `yang-mills-group/OpenAI-YM-20261028` (Apache-2.0), `YM/RG/History.lean` (:81 `monitor`, :83 `threshold`, :84 `allowance`, :85 `forcedAllowance`, :213 `Conv`, :223 `Transfer`)
Clipped action	Source	`YM/RG/PathActions.lean` (:145 `kink`, :218 `clipped`)
Gas of activities	Source	`YM/RG/Activities.lean` (:98 `EBound`, :135 `gas_one`)
Cells and seeds	Source	`YM/Pressure/Seeds.lean` (:48 `SliceSet`, :51 `Plot`, :120 `Serves`, :124 `cash`, :129 `Grants`), `YM/Push/SECTOR_MEASURE/Cells.lean:51` (`Seeded`), `YM/Next/FLUCTUATIONS/Grants.lean:24` (`grants`)
Positive remainder for intermediate coarse plaquettes	Source	`YM/Bid/Energy.lean` (`Crowds` at :94–:97, :101 `E05`)
Conversion between the two ledgers	Source	`YM/PCalc/Band.lean` (:49 `Move`, :92 `holding`), `YM/AnalyticBridge/VERIFIED_RESET_STEP/Opening.lean:19` (`late_force`), `YM/Next/SCHEDULE/Salary.lean:178` (`saved`), `YM/AnalyticNext/CONNECTED_ACTIVITY_JETS_E6/Bulkhead.lean:78` (`dictionary_fit`), `OptionCost.lean:83` in the same directory (`costs`), `Issue.lean:73` in the same directory (`Invoices`), `YM/FullVolume/C/Songbook.lean:76` (`Roof.Lyrics.supplier`)
The finest step	Source	`YM/Extra/WILSON_STARTUP/Ownership.lean:68` (`due_eq`)
That the assessment contains no description of the large-field treatment	Assessment	Full-text search of the assessment (`docs/assessment-1.pdf` in `yang-mills-group/YM`, 2026-11-10) (the words large field, small field and polymer do not occur)
This book's study's side	Assumption	The "large-field estimate" and "large-field gas" in the list of premises (28-04), and the open places (28-06)

Terms used in this chapter — large field, small field, threshold, clipped action, activity, gas of forced activities, flag, seed, cell, partition of unity, energy per seed, ledger, one-step certificate, stamp, calendar, persistent defect. Their meanings are in the A-02 glossary.

Next chapter: 14-05 Ideas this book's study lacked — we list 35 ideas, including the treatment in this chapter, divided into groups.

2026-11-13 · chapter Yang–Mills and the mass gap Part 14 Comparison — this book's Lean, and ideas this book's study lacked list of ideas · Kiichi, Shiori and Rin

Ideas this book's study lacked — a list in seven groups

> This chapter is this book's own investigation. The "What it does" column of each item is established knowledge (a reading of the published construction), and the "What changes" column is inference.

What this chapter shows — Reading the published construction, we pick out 35 ideas that this book's study lacked and list them in seven groups. Each item is written in three columns: what it does (with its location in the source), what this book's study did instead, and what changes. At the end we place the five things that pointed in the same direction, and cautions for reading this list (what cannot be said).

Prerequisite chapters — 14-01 Comparing skeletons to 14-04 Large fields. Detailed explanations of each item are in the chapters of Parts 9–13.

Said first — The "What changes" column is an inference applied to this book's study; it has not been checked that the idea can be used as is in this book's design. Every idea works in the published construction in combination with other components, and taken out alone it need not have the same effect. Whether each item is among the dependencies of the main theorem has been checked only to the extent written in §09.

The order of this chapter

1. The one-step design — probability kernel, projected predictor, gauge fixing in two strands, growing sides
2. The small-field step — exact identities, finite-range covariance, derivative norms, redefining the ledger
3. Large fields — clipping the representation, re-counting, two ledgers, rarity on the action side
4. The coupling — a coefficient in the representation, the shooting method, staying at weak coupling
5. How limits are taken — renormalisation as a tool for estimates, Cauchy at a fixed step, uniqueness, the root seminorm

6. The infrared and reconstruction — the trace representation, phase decomposition, Ward identities, the trace anomaly, fields as a quotient
7. How the Lean is organised — request propositions, bundling over one construction
8. What pointed in the same direction — five things
9. Cautions for reading this list — what cannot be said

Labels: Source = we read the source of the published construction; Inferred = this book's inference; Assessment = from the committee's assessment. The other labels (Proved here, Cited, Reading, Lean and so on) mean the same as in the rest of the book (details in "Labels" in 6-00). For Lean theorems of this book's study, the name and scope are written on the same line, outside the labels.

01

The one-step design

Details are in 14-02 and Part 9.

Idea	What it does (source)	In this book's study	What changes
Make one step a probability kernel rather than a map	Draw the coarse links with Gaussian-type fluctuations $e^{-\beta c_{\mathrm{pen}}\,\mathrm{price}}$ around the predictor. The penalty coefficient c_{pen} is any positive number (`kernel.penalty` in the source's wording) (`YM/RG/Block.lean:71` `density`)	Push forward by a deterministic block map (19-04)	The constraint is no longer hard, and c_{pen} enters the precision of the free field of one step. In exchange, the push-forward measure becomes an object that depends on β
The predictor is an explicitly built central value projected onto the group	The group element nearest to the weighted average of transports along parallel straight lines; if the spread is large, one line is drawn at random (`YM/RG/Patch.lean:159`, :171, `YM/Branch/Center.lean:72`)	Bring the average back onto the sphere (retract). The background field is the minimiser of a constrained action (28-05)	The background minimisation problem need not be solved inside the definition of one step, and the value is always in the group

Idea	What it does (source)	In this book's study	What changes
Gauge fixing in two strands, drawn at random and summed over	Symmetric minimisation for small fields; for large fields, the root and order of an axial gauge drawn uniformly. Through the sum over labels, exactly invariant under the hypercubic group and coarse translations (`YM/RG/Patch.lean:102`–:155, `YM/Branch/Covariant.lean:50`)	Symmetric averaging over a family of paths. Gauge at each step chosen as Landau, axial or tree (28-05)	Gauge fixing is determined even for large fields, and the lattice symmetry is not broken
Let the side grow with the step	The side is an odd number at least 3 (`YM/Lattice/Scale.lean:28`), non-decreasing (`Fits.monotoneL`), with $\log b\, m / \log D_m \to 0$ (`Fits.mesh`, `YM/RG/History.lean:97`–:100). The one-step error is left at $O(1)$ and removed by dividing by $\log b$	Side fixed at 2 (and also in the side-3 design)	Precise two-loop estimates become unnecessary. In exchange, the sequence of lattice spacings becomes a special one, and there is no universality with respect to the choice of sequence Assessment

02

The small-field step

Details are in 11-09.

Idea	What it does (source)	In this book's study	What ch
Link steps by exact identities without approximation	The field of the one-step certificate is the push-forward identity (`ident` at `YM/PCalc/Band.lean:50`). Both the error of the Gaussian approximation and the non-Gaussian remainder are carried inside the integrand	The loss of margin in handing over was carried on paper as a loss in the estimates	They ca carried terms o integra rather t losses. T ledger s consists inequal so this c not mea estimat no approxi

Idea	What it does (source)	In this book's study	What ch
Replace the covariance by an approximate inverse cut off at finite distance	Invert the Gram matrix only inside a window and symmetrise; the difference from the true inverse is exponentially small in the window radius (`YM/SectorNext/GAUSSIAN_SECTOR_CHANGE/Windows.lean:184` `windows`). The difference is kept inside the identity	A covariance decomposition using contraction uniform in the step (15-12)	Indepen at a dist holds ex giving a foundat expansi
Measure the size of activities by norms of finite-order derivatives	The supremum of finite-order derivatives in directions of size t (`YM/RG/Masks.lean:164` `supBounds`)	Estimates on paper measured by an analyticity radius	Losses c type wh radius s at every do not c
Carry agreement of the main action up to fourth order as a condition	The condition that the coarse main action and the stationary value of the fine main action plus the constraint penalty agree, at flat configurations, in their jets up to fourth order (`YM/PCalc/Chart.lean:108` `ClassicalMatch`). It is closed by `sailsW` (`YM/FullVolume/Roll/Fiscal.lean:97`). The coarse main action is data given separately	The expansion of the background field was placed on the premise side (28-04)	The remainc become small en to fit in ledger Inferred
Redefine the ledger at every step from that step's coupling	The threshold and ledger are functions of the reference coupling $\hat{\beta}_m$ at step m (`YM/RG/History.lean:83`, :84). One shows only that the exit fits into the fixed ledger of the next step	The next step's estimates were derived from the previous step's, with the risk that losses accumulate step by step	The ind hypothe carries losses. I from rereadi prepaid step cor
Do not leave marginal terms in the activities; move them to the coupling and the vacuum term	Flat values go to the vacuum term and quadratic parts to the coupling; the small-field activity ledger is $\hat{g}^{11/5}$ (`YM/RG/History.lean:84`). The exponent lies between the marginal order 2 and the first non-marginal order 3	—	How ma orders r be matc for the remainc fit in the ledger c counted order

03

Large fields

Details are in 14-04 and Part 11.

Idea	What it does (source)	In this book's study	Wha
Clip the main part of the representation	Represent the action of plaquettes beyond the threshold weakened to linear growth, and put the difference into the activities (`YM/RG/PathActions.lean:218`). Steps are linked by identities, and the measure does not change	Hand over the quadratic action as the main part	It b foot sett ever inec con sati sect con
Keep no history of regions; re-count at each step	The induction hypothesis has no history field, and seeds are re-counted at each step (`YM/RG/History.lean:213`)	Large-field regions stacked up and fixed thereafter	Pers defe trea step step
Keep two ledgers	A weak ledger $\exp(-p^{1/5})$ in the induction hypothesis and a strong bound $\exp(-gp^2)$ in the one-step certificate (`YM/PCalc/Band.lean:49`), linked by a one-line inequality (`YM/Next/SCHEDULE/Salary.lean:178`)	Thought in terms of one ledger (of type e^{-c/g^2})	The esti put indu hyp wea rebu step unif con disc the side
Put rarity on the action side rather than in the activities	Keep the clipped action explicitly in the representation (`YM/RG/PathActions.lean:218`), and make the activities factors carrying the difference. Rarity is extracted from the action difference at each step Inferred (line of reading in 11-06, 14-04 § 07)	Rarity carried over as estimates on activities	No a for is n
Split clusters by whether they contain a coarse flag	Clusters containing one become large-field activities at the next step; those without go to the small-field side (`ThickP`, `ThinP`, `YM/AnalyticNext/CONNECTED_ACTIVITY_JETS_E6/Colony.lean:26`, :27)	A separate case analysis for defects returning to small fields	One suff
Use $\exp((\log\log\beta)^2)$ in the threshold	Threshold $t = \hat{g}\,p$, $p = \exp((\log\log\hat{\beta})^2)$ (`YM/RG/History.lean:81`, :83)	A threshold that is a polylogarithmic multiple of the coupling	Poly loss over with cou pow

Idea	What it does (source)	In this book's study	Wha
Cells with hysteresis and a smooth partition of unity	Cells for each seed type glued by a partition of unity; the conditions are imposed globally while the estimates are localised around seeds (`YM/Push/SECTOR_MEASURE/Cells.lean:51`, `YM/Pressure/Seeds.lean:51`)	Cut with small-field characteristic functions	Disc at th bou avoi eacl esti inde
Locate the point of taking large sides in the coercivity constant	First-order quantities of fine plaquettes shrink onto coarse plaquettes with weight L^{-2}, so taking the side large lets the fine side win Inferred (`E03`, `E06`, `E07` in `YM/Bid/Energy.lean`, :67, :110, :130). The lower bound on sides is a calendar field (`YM/Price/Intent.lean:39` `Large`)	The point of taking large sides was the gain in coupling	The side proc con larg inec Infe

04

The coupling

Details are in 14-03 and Part 10.

Idea	What it does (source)	In this book's study	What changes
Define the coupling as a coefficient in the representation, and reread it	The step coupling is the coefficient of F^2 in the representation, with two normalisations forbidding reassignment (docstring at the top of `YM/RG/History.lean`). It is reread from the log-ratio of the density at two configurations and monotonicity (`YM/UVNext/COUPLING_RESET/Reset.lean:50`)	Match with a quantity determined by the measure alone (19-07). No representation	One can follow directly in the representation how the coupling moves by its one-loop value at each step
The shooting method	Build nested intervals of bands at each step and choose the bare coupling by the intermediate value theorem (`YM/FullVolume/Arc.lean:95`). Only continuity and the one-step error inequality are needed	The intermediate value theorem from global endpoints ($\beta = 0$ and $\beta \to \infty$)	One can match even quantities whose endpoint values are unknown. Under the calendar condition that the sum of one-step errors fits within a small fraction of the band width, it works in the same form regardless of the number of steps

Idea	What it does (source)	In this book's study	What changes
Carry every step at weak coupling	$\beta_m \in [\hat{\beta}_m/2, 2\hat{\beta}_m]$ at every step (field `band` at `YM/RG/History.lean:213`). The reference coupling is taken large and one stops at the step with spacing 1	Thought of weak to strong coupling as one flow	The strong-coupling regime is not passed through by renormalisation. Beyond that (long distances) is handled by compactness

05

How limits are taken

Details are in the second half of Part 11 and in Part 12.

Idea	What it does (source)	In this book's study	What chang
Renormalisation is a tool for estimates, not the object of the limit	What is taken to the limit are correlations of renormalised local fields under the measure of the original lattice Assessment (Assessment § 4.3). Reflection positivity is used only on the original lattice (`YM/Slice/Reflection.lean:98`)	Take the limit of the push-forward measures themselves (28-01)	Reflection positivity c the pushec forward measure is needed (14 § 07)
Show Cauchy at a fixed coarse step first, then remove the step	The effective density at a fixed step is Cauchy uniformly in the cutoff (`YM/FullVolume/Fields/Cement.lean:50`); bound the difference between the true correlations and the read-out at step j by a quantity independent of N (`Sever.echoes`, `YM/FullVolume/Fields/Peat.lean:42`); conclude that the true lattice correlations form a Cauchy sequence in N (`Rising.arrives`, `YM/FullVolume/Fields/Timber.lean:74`). The limiting object itself is not built	Subsequential limits	Subsequer become unnecessa Convergen follows by continuou operations from the ingredient the representa being Cauc
Derive convergence of the ingredients from uniqueness rather than quantitative estimates	The coupling is anchored at step 0, and the activities shrink by 1/16 per step. There is only one orbit satisfying the conditions in both directions. Compactness and the subsequence principle give convergence of the full sequence (`YM/FullVolume/Glide/Triumph.lean:37`, `YM/FullVolume/Glide/Coincide.lean:58`). No rate comes out	Had an implication of the same type for orbits of an abstract skew product (28-02), but the map was not defined from the lattice	No quantitativ estimate comparing cutoffs N a N' is neede

Idea	What it does (source)	In this book's study	What chang
Differences at the fine end need only be bounded	Differences in the initial values of the activities are forgotten by $(1/16)^k$ as one goes down the steps (`YM/FullVolume/Glide/Wings.lean:59` `culprit`)	It existed in abstract form: in the implication for skew-product orbits, the limit does not depend on the choice of bare irrelevant coordinates (`SkewOrbit.limit_independent`, 28-02). The map was not defined from the lattice	The representa at the fines step need r be made sr
Make the one-step ledger independent of the volume	The ledger is a sum per point (`YM/RG/Activities.lean:98` `EBound`), and the contraction constants do not depend on the volume. Estimates at the level of densities and partition functions contain exponentials of the volume; the uniform estimates handed to infinite volume are built, with the volume-uniform ultraviolet estimates (`Wcmain` and others) as input, from reflection positivity on the original lattice, the chessboard and the transfer-operator representation Assessment (Assessment § 5)	Forms uniform in volume and step were placed inside the premise statements (28-04)	The divisic of labour between estimates becomes c
Give the renormalisation of composite fields by the inverse of the renormalisation transport	Have at each step a linear isomorphism on a finite-dimensional space of coefficients (with uniformly bounded inverse), and carry from the reference step by the inverse (`Club` at `YM/FullVolume/Mark/Nursery.lean:11`, `YM/FullVolume/Mark/Sages.lean:66` `spends`)	The endpoint is the coarse-lattice measure; no calibration of observables	Correlation of local fie can be the endpoint
State the estimates handed to compactness in the root seminorm of the chessboard	The size of renormalised observables in the root seminorm is at most $A\, a_m^{-d-\varepsilon}$, uniformly in the volume (`YM/Extraction/Center.lean:38` `Wcmain`, `YM/Cells/Tiling.lean:226` `rootTest`)	The chessboard is used for rarity of large plaquettes (22-01)	One of the estimates handed fro the ultravi to the infra can be wri as a single volume-uniform statement (there are other inpu from the ultraviolet 03)

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06
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The infrared and reconstruction

Details are in Parts 12 and 13.

Idea	What it does (source)	In this book's study	What changes
Obtain equicontinuity from the trace of the transfer operator	Write expectations as traces of the transfer operator, and bound the movement of insertions as reassigning gaps, by Hölder. Arzelà–Ascoli applied to correlation functions rather than measures (`YM/Extraction/Lipschitz.lean:166` `first_surface`)	Compactness in weak convergence of measures	Uniform estimates with derivatives on correlations at separated points are obtained
Without proving clustering, split into phases and then reconstruct	Split into phases by the Gelfand representation of a commutative algebra of operators compressed to the vacuum space, and obtain uniqueness of the vacuum in each component (`YM/Phase/Mixture.lean:127`, `YM/EveryPhase/Vacuum.lean:146`). Commutativity comes from Euclidean invariance	Prove clustering in the strong-coupling window (20-03)	One can get as far as reconstruction without proving uniqueness or decay
Lift rotational invariance from the Ward identities of the stress tensor	Ward identities for translations and rotations (`translation`, `rotation` at `YM/Euclid/Stress.lean:90`–:97). That rotational invariance is not spontaneously broken in the limiting state is derived, from the stress flux vanishing fast far away, by an argument using reflection positivity in several orthogonal planes Assessment (Assessment § 6.3)	Not treated	One route from the discrete lattice symmetry to continuous rotational invariance is fixed
State non-freeness by the logarithm of the trace anomaly	The short-distance law for the two-point function of the trace, the limit of $r^8 \log^2(1/r)$ (`ChallengeSpec.lean` :145–149 and :173–174)	Not treated	"Non-trivial" can be stated by the value of a single limit. The types of free field excluded are limited (13-02)
Define the field spaces as a quotient by the null space of correlations	Quotient by the part where all correlations at separated points vanish (`YM/Euclid/Null.lean:34` `null`)	Not treated	A form that could also be used to define continuum objects for non-local observables (loops, twists) Inferred

07

How the Lean is organised

Idea	What it does (source)	In this book's study	What changes
Conditional theorems with request propositions as hypotheses, all closed at the end	Intermediate theorems are written in conditional form taking a request proposition (`Bid`) as hypothesis, the proofs (`Work`) are placed separately, and the theorem above hands over all the answers (`YM/RG17/Joint.lean:83`–:85)	Implications with hypothesis fields (28-01, 28-02). The same type	The difference is that at the end all requests are closed
Bundle all conclusions over one construction chosen once	To avoid the accident that objects whose existence was shown separately are incompatible, the choices are gathered into one run (docstring at the top of `YM/Org/Input.lean`, around :14)	Objects were taken separately for each theorem	The types guarantee that the conclusions are about the same objects

08

What pointed in the same direction

1. Fixing the scale by matching a quantity determined at unit length to a fixed value (14-03).
2. Odd sides, with blocks centred on coarse points. Symmetric averaging (14-02).
3. Fixing the volume first, then going to infinite volume by compactness (14-01).
4. Using reflection positivity and the chessboard as the source of volume-uniform estimates (14-01).
5. The one-loop coefficient. With conventions matched, it agrees with the value for one side-2 step in this book `Inferred` (14-03 § 07).

09

Cautions for reading this list

Please do not reread the items of the list in the following ways. Each would say more than the statements in the source.

- One cannot say "because the representation is clipped, the premise of persistent defects no longer applies". Steps are linked by identities, and the measure does not change. What can be said is only that the main part of the representation is clipped, the difference is put into the activities, and an inequality is set up for every configuration satisfying the section and cell conditions.
- One cannot say "the problem of estimates uniform in the number of steps does not arise". The weak ledger is also a function of the step, and the step-uniform conditions are discharged on the calendar side.

- One cannot say "the smallness of incoming activities is not used". It does not enter the formula for the exit rate, but estimates of incoming activities are used on the side of the one-step hypotheses to make the sums converge.
- One cannot say "the coarse main action is determined as the expansion of a stationary value". Agreement up to fourth order is carried as a condition, and the coarse main action is data given separately.
- One cannot say "volume-uniform estimates come only from reflection positivity on the original lattice". The one-step ledger has a volume-independent form, and the input from the ultraviolet to infinite volume is not just one.
- One can say "used by the main theorem" only for declarations whose chain to the main theorem has been checked. What this book's study checked are the path of shooting and the one-step theorem (from `rodB` at `FullVolume/Stage/Rods.lean:26`), convergence in the cutoff (`allB` and others), `sailsW` and `realUV`. Many of the large-field lemmas are tied in the record of how declarations are used to other headline theorems, and we have not checked whether they lie on the path of the main theorem.

What this chapter can say

Can say	Cannot say
Source The "What it does" of each item comes from reading the definition or statement of the declaration at its location	The content of the proofs. Whether they are among the dependencies of the main theorem has been checked only in part
The columns on this book's study's side are based on the theorems and the list of premises of Parts 15, 19, 22 and 28	—
Inferred The "What changes" column	That the idea can be used as is in this book's design

Sources and reproduction

Item	Kind	Source / tool
Locations of each item	Source	Repository `yang-mills-group/OpenAI-YM-20261028` (Apache-2.0). Paths and lines are in each row of the tables; the lines were checked by this book's study by opening the source
Definition of the correlations, order of limits, absence of universality	Assessment	The assessment (`docs/assessment-1.pdf` in `yang-mills-group/YM`, 2026-11-10), § 4.3, § 5, § 6

Item	Kind	Source / tool
This book's study's side	—	The chapters cited in each row (19-04, 15-12, 22-01, 28-01, 28-02, 28-04, 28-05, 28-06 and others)

Terms used in this chapter — probability kernel, predictor, projection onto the group, gauge fixing in two strands, growing sides, finite-range approximate inverse, finite-order derivative norm, ledger, clipped action, seed, stamp, calendar, shooting method, anchor, forgetting of the upper end, root seminorm, phase decomposition, request proposition. Their meanings are in the A-02 glossary.

Next chapter: 14-06 What this book has and the published construction does not — the comparison in the other direction, and a bridge to Volume 3.

2026-11-13 · chapter Yang–Mills and the mass gap Part 14 Comparison — this book's Lean, and ideas this book's study lacked only in this book · Kiichi, Shiori and Rin

What this book has and the published construction does not — a bridge to Volume 3

> This chapter is this book's own investigation. Descriptions of the published construction are established knowledge (a reading of the published construction), and § 09 is a map of open questions.

What this chapter shows — We list statements of kinds not among the claims of the published construction that are closed in the Lean of this book's study, in four groups (the strong-coupling lattice, centre vortices, rarity of large plaquettes at weak coupling, the fermion weight). We list separately the three things that can be called specific to this book's renormalisation design. We tabulate where each stands relative to the open questions of the published construction (convergence and uniqueness, the mass gap, Wilson loops and the area law), and at the end give an outline for going from where the published construction stops (the step with spacing 1, weak coupling) on to Volume 3.

Prerequisite chapters — 14-01 Comparing skeletons, 6-04 what has not been shown, the chapters of Parts 20–22.

Said first — Every theorem in this chapter is a formalisation of a result of a known type. The lattice spacing is fixed at 1, and nothing is said about the continuum limit. Most are statements inside a strong-coupling window: $18 \tanh |\beta| < 1$ ($|\beta| < 0.0556$ Reading), the 441 version of the area law and vortex free energy ($\beta < 1.135 \cdot 10^{-3}$ Reading), and the 4096 version ($\beta < 1.22 \cdot 10^{-4}$ Reading). They are statements at the opposite end (strong coupling, fixed lattice spacing) from the questions the published construction left open; they neither answer those questions nor point towards them. What can be said is only that we have statements of the same kind of question, at a different place on the lattice.

The order of this chapter

1. Conventions of this chapter — known types, fixed lattice spacing, nothing said about the continuum limit
2. The strong-coupling lattice — infinite-volume limit, mass gap, clustering

3. The area law at strong coupling, and an upper bound on the vortex free energy — inside the window, the rate per area does not depend on the lattice size (the prefactor remains)
4. Centre vortices — the thin-vortex floor, the thick-vortex floor, pairs of Polyakov loops, the vortex ratio
5. Rarity of large plaquettes at weak coupling — uniform in volume, explicit threshold
6. The fermion weight — reflection positivity, positive partition function, symmetries
7. What is specific to this book's design — at most three. What neither has
8. Relation to the open questions (Table 1) — the same kind of question, at the opposite end
9. A bridge to Volume 3 — how to turn what lies beyond the step with spacing 1 into questions

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Labels: Source = we read the source of the published construction; Inferred = this book's inference; Assessment = from the committee's assessment. The other labels (Proved here, Cited, Reading, Lean and so on) mean the same as in the rest of the book (details in "Labels" in 6-00). For Lean theorems of this book's study, the name and scope are written on the same line, outside the labels.

01

Conventions of this chapter

The assessment lists five open questions as the main points the published construction does not treat Assessment (Assessment § 8).

1. Prove convergence rather than subsequences, and uniqueness of the infinite-volume limit.
2. Define the probability measure of the continuum theory.
3. Prove a uniform mass gap (exponential decay of correlations and a spectral gap of the Osterwalder–Schrader Hamiltonian).
4. Construct Wilson-loop observables and relate them to local observables. Prove the area law for Wilson loops.
5. Construct one-particle states and study the energy–momentum spectrum.

The Lean of this book's study has statements of the same kind as 1, 3 and 4 of these. But their place differs in three respects.

- **Known types.** Every theorem is a formalisation of a result of a type known in lattice gauge theory (strong-coupling expansion, the Osterwalder–Seiler area law, 't Hooft's twists and the vortex free energy, the chessboard Peierls argument)Cited .
- **Fixed lattice spacing.** Each is a statement on a single lattice with lattice spacing fixed at 1, or about its infinite-volume limit. Nothing is said about the continuum limit.
- **Coupling regime.** Most are statements inside a strong-coupling window. The window differs from theorem to theorem: $18\tanh|\beta| < 1$ ($|\beta| < 0.0556$Reading), $24(1 - e^{-4|\beta|}) < 1$ ($|\beta| < 0.0106$Reading), the 441 version of the area law and vortex free energy $441(1 - e^{-2\beta}) < 1$ ($\beta < 1.135 \cdot 10^{-3}$Reading), and the 4096 version ($\beta < 1.22 \cdot 10^{-4}$Reading). The published construction carries every step at weak coupling, so the two do not overlap in the coupling. These are statements at the opposite end from the questions the published construction left open.

02

The strong-coupling lattice

For the SU(2) Wilson measure on the periodic torus $(\mathbb{Z}/N)^4$, the following are in Lean in strong-coupling windows (Part 20).

- **Infinite-volume limit (without subsequences).** For $18\tanh|\beta| < 1$, the sequence of torus measures lifted periodically converges weakly, without taking subsequences, to a single probability measure ν_β. The limit is the same for every sequence of sizes. Lean: `InfVol.wilson_infvol_unique`, `AllN.wilson_infvol_allN` (20-03).
- **Translation invariance and clustering.** The limit is invariant under unit translations in the four directions, and correlations of distant observables decay exponentially. Lean: `TransI.wilson_infvol_translation`, `WilsonLimit.cluster_limit`.
- **Mass gap.** The transfer operator T on the reconstructed Hilbert space satisfies $\|T\| \le e^{-m}$ ($m > 0$) on the orthogonal complement of the vacuum, and that complement is non-zero (non-degeneracy). The form for some limit point is in the Dobrushin window (`WilsonLimit.wilson_massGap_dob_exists`), and the form for all limit points is in the window $24(1 - e^{-4|\beta|}) < 1$ (`WilsonLimit.wilson_massGap_nondegenerate`). There are also a form for the gauge-invariant sector and a statement combining everything (`GaugeZ.wilson_massGap_gauge_exists`, `InfMain2.wilson_infvol_main2`, 20-04).

What is not said: outside the window, other boundary conditions, finiteness of the mass (the bound $\|T\| \le e^{-m}$ is only a lower bound on the mass), a mass gap in the continuum limit.

03

The area law at strong coupling, and an upper bound on the vortex free energy

Area law (finite box). In SU(2) Wilson lattice gauge theory on $(\mathbb{Z}/N)^4$, when $0 \le \beta$, $q = 1 - e^{-2\beta}$, $4096q < 1$ and $1 \le R, T < N$, the Wilson loop W on the boundary of the $R \times T$ rectangle in the $(0,1)$ plane with a corner at the origin satisfies

$$\left|\langle \tfrac{1}{2}\operatorname{tr} W\rangle_\beta\right| \;\le\; \frac{16\,(4096\,q)^{R\cdot\min(T,N-T)}}{4096\,(1-4096\,q)} \quad (1)$$

The constants do not depend on N, R, T. Lean: `AreaLaw.area_law_wilson` (the form with exponent RT for $2T \le N$ is `area_law_wilson_area`). Rectangles with any corner and orientation are `area_law_wilson_general`. The versions with constants tightened to 6 and 441 (`area_law_wilson_sharp`, `area_law_wilson_max_sharp`) hold in the window $441q < 1$ ($\beta < 1.135 \cdot 10^{-3}$ Reading), with exponent $\max(R \cdot \min(T, N-T),\, T \cdot \min(R, N-R))$.

Area law (infinite volume). Under the three conditions $\beta > 0$, $18 \tanh \beta < 1$ (the Dobrushin condition) and $4096q < 1$, in the strong-coupling infinite-volume limit $\nu\beta$, for rectangles in the $(0,1)$ plane with a corner at the origin and all $R, T \ge 1$,

$$\left|\langle \tfrac{1}{2}\operatorname{tr} W(\partial(R\times T))\rangle_{\nu\beta}\right| \;\le\; \frac{16\,(4096\,q)^{RT}}{4096\,(1-4096\,q)} \quad (2)$$

holds. Lean: `AreaLaw.area_law_infvol` (at every limit point, `area_law_limit_sub`, 20-06). Both are formalisations of a known type (Osterwalder–Seiler) Cited .

Upper bound on the vortex free energy. On $(\mathbb{Z}/N)^2 \times (\mathbb{Z}/M)^2$, for the twisted partition functions Z_0, Z_1 (19-02), for every background, when $\beta \ge 0$ and $441q < 1$,

$$\frac{|Z_0 - Z_1|}{Z_0 + Z_1} \;\le\; \frac{N^2\,(441\,q)^{M^2}}{441\,(1-441\,q)} \quad (3)$$

holds. Lean: `VortexStrongMN.vortex_floor_MN_sharp` (the only hypotheses are $\beta \ge 0$, $441q < 1$ and the choice of background). The forms for the isotropic torus $(\mathbb{Z}/N)^4$ are `VortexStrong.vortex_floor` (hypotheses $\beta \ge 0$, $4096q < 1$, constant 4096 on the right-hand side, exponent N^2) and `VortexStrong.vortex_floor_sharp` (the 441 version).

Inside the strong-coupling window, the rate per area $-\log(441q)$ does not depend on N, M. The prefactor N^2 (the transverse area) remains. Outside the window there is no form uniform in the transverse size. The sign side, $Z_1 \le Z_0$, is not stated.

04

Centre vortices

For the vortex ratio $\theta_M = (Z_0 - Z_1)/(Z_0 + Z_1)$ and its relatives, the following are in Lean (Part 21). None uses the block map; they use only the fine-lattice Wilson measure and the twisted partition functions.

Item	Content	Lean and scope
Thin-vortex floor	$\theta_M \le \tanh(N^2\lvert\beta\rvert)^{M^2}$ (also the two-sided form)	`TwistedWilson.twisted_vortex_bound`, `_abs`. All real β, no hypothesis fields (21-02). The rate depends on N
Thick-vortex floor	For each sufficiently large β and sufficiently large M, a floor with the minimal vortex action S_N in the exponent	`TwistBlocks.thick_vortex_floor` (no hypothesis fields, 21-03). The value of S_N and the thresholds are not explicit
Minimal vortex action	$0 < S_N \le 2N^2$. $S_N < 2N^2$ for $N = 2, 3, 4$	`VortexPos.SN_pos`, `TwistBlocks.SN_bounds`; for $N = 2, 3, 4$ the computer certificates `VortexCert.SN_two_lt`, `SN_three_lt`, `SN_four_lt` (21-05)
Sign of θ	If the vertical size M is even, $Z_1 \le Z_0$ for all real β	`HalfSplit.Z1_le_Z0`, `TwistBlocks.theta_nonneg` (hypothesis: M even, 21-06)
Pairs of Polyakov loops	$\lvert\langle P_b(0)P_{b'}(T)\rangle_0\rvert \le 2\tanh(N^2\lvert\beta\rvert)^{M\min(T,M-T)}$ $(0 \le T \le M)$ and others	`PolyakovT.polyakov_pair_all` (hypothesis $M \equiv 0 \pmod 4$, all real β), `HalfSplit.polyakov_pair_theta`. No tension uniform in N is stated
The vortex ratio — continuity, limit, matching	θ_M is continuous, $\theta_M(0) = 0$, tends to 1 as $\beta \to \infty$, and has a largest solution for every $\kappa_0 \in [0, 1)$	`ThetaTuning.thetaM_continuous`, `ThetaLimit.tendsto_theta_one`, `ThetaLimit.tuning_all` (21-07)
Lower bound on the defect of a twisted box	A box with twists in two orthogonal planes has no flat configuration, and the defect of a symmetric box is at least $1/2$ independently of N	`TwistFlat.no_flat_twist11`, `TwistFloor.defect11_sym_half` (21-07)

What is not said: the continuum limit, rates uniform in N outside the strong-coupling window, confinement itself. That a twisted box has no flat connection is of a known type Cited .

05

Rarity of large plaquettes at weak coupling

For the Wilson measure on a torus of even side, when β is large (weak coupling), the probability that $\operatorname{Re} U_p < 0$ on every plaquette of a set X of plaquettes is at most $q_1^{\lfloor |X|/6 \rfloor}$, where $q_1 < 1$ does not depend on X or the size of the torus. Lean: `FoldCap.rarity_explicit` (explicit threshold); the form with rate $e^{-c\beta}$ is `FoldQuarter.rarity_uniform4` (22-01). A formalisation of the chessboard Peierls argument from reflection positivity Cited .

This is an estimate of the type that controls large plaquettes (large fields) at one scale, and it stands in a position facing this book's study's large-field question (14-04, the large-field estimate in 28-04) Inferred . It is not a statement of the type that gives uniqueness. What is not said: measures at coarse scales, a form spanning steps, a form uniform in the lattice spacing. The threshold is a value convenient for the proof, far from the band of β of physical interest.

06

The fermion weight

For the lattice measure with fermions, weighted by the determinant of the Wilson–Dirac operator, Lean has reflection positivity in lattice-point planes (`WilsonGauge.site_reflection_positive_wilson_full`), positivity of the partition function (`WilsonGauge.partition_pos_wilson_full`), and invariance under generators of the lattice symmetries (`CubicSym.wfProb_lattice_generators` and others) (20-05). The published construction is pure Yang–Mills (without fermions) Assessment . What is not said: the continuum limit, chiral symmetry, mass renormalisation, a mass gap with fermions.

07

What is specific to this book's design

On the renormalisation side, at most the following three can be called specific to this book's design. All are statements at the level of design.

1. **A fixed block map independent of the cutoff.** This book's block map is a single map depending on neither the coupling nor the cutoff, and consistency between steps is a push-forward identity (for any family of fine measures, pushing one step at a time or all at once gives the same measure). Lean: `ChainBlockMap.hcons_blockMap` (19-04). It is an identity coming from composition of maps, a statement of thin content. The published construction's kernel depends on $\beta(N, p, m+1)$, and the map changes with the cutoff Source (14-02).

2. **Reflection positivity of the pushed-forward measure.** Reflection positivity of the Wilson measure pushed forward by the side-3, centred, uniform map, on periodic lattices with L even, for reflection in a lattice-point plane, in the class of bounded measurable functions depending only on links strictly between two planes of coarse lattice points (gauge invariance not imposed; all real β; no version for planes through link midpoints). Lean: `Block3.rp_push` (14-02 § 06). The published construction's statement of reflection positivity concerns only the original Wilson measure Source (`YM/Slice/Reflection.lean:98`, within the range of a search by name).
3. **Matching by quantities determined by the measure.** Existence of sequences of couplings matching to a fixed value the expectation of the average of coarse plaquettes under the push-forward measure (`Block3.tuned3_exists`) or the vortex ratio (`ThetaLimit.tuning_all`) (14-03). The hypotheses of `tuned3_exists` are only design values and arithmetic ($0 < r_0 \le 1$, $3^{m0} L \ge 2$, target $\kappa_0 \in [0, 1)$); `tuning_all` speaks lattice by lattice (with N, M fixed) about $\kappa_0 \in [0, 1)$. Neither states monotonicity, divergence or boundedness of the sequence of couplings, or uniqueness of the solution. In the current design (side 2), existence of a matched sequence is conditional. The published construction's matching uses a coefficient in the representation, so they differ already at the level of design choices.

What both have, or neither has. We do not write the following three as "only in this book".

- A limiting measure on a fixed coarse lattice. In this book's study it exists as a subsequential limit (28-01), and for the published construction's kernel it would presumably follow from the uniform convergence of densities at a fixed step (`Flux.fanned`, `YM/FullVolume/Fields/Cement.lean:50`) Inferred. The published construction merely has no named statement for it Source (within the range of a search by name).
- The second open question of the assessment is the probability measure of the continuum theory, and a family of measures on a fixed coarse lattice is not that Assessment.
- Neither has continuum Wilson loops. This book's area law is a statement about lattice Wilson loops.

08

Relation to the open questions

The infrared side of the published construction is closed with upper bounds only Source. Infinite volume is taken along subsequences, and no statement of exponential

decay of long-distance correlations was found Source (within the range of a search by name). What there is at long distances are polynomial upper bounds uniform over the phases (spectral weight at energies $(0, u]$ of order u^2, `YM/EveryPhase/Light.lean:221`; an estimate τ^{-2} in heat time τ, `YM/EveryPhase/Decay.lean:173`) Source . This form is also compatible with a massless theory Inferred . The continuum correlations themselves exist at all separated points Assessment (Assessment § 5). Parallel transport along lattice paths (`travel` at `YM/Lattice/Path.lean:105`) exists, but no continuum Wilson-loop observables were found Source (within the range of a search by name). The boundary conditions are periodic only (`YM/Wilson/Law.lean:87`) Source .

Table 1 Items of this book's study and their relation to the open questions of the published construction

Item of this book's study	Open question of the same kind	What is the same	What differs
Strong-coupling infinite-volume limit (without subsequences)	1. Convergence and uniqueness	The sequence of periodic lifts has a single limit without subsequences	Opposite end (strong-coupling window, lattice spacing 1). The published construction's limit is of continuum correlations coming from the weak-coupling ultraviolet
Strong-coupling mass gap, clustering	3. Uniform mass gap	A gap of the transfer operator, and exponential decay of correlations	As above. Nothing about a gap in the continuum limit
Strong-coupling area law	4. Wilson loops and the area law	Expectations of lattice Wilson loops are exponentially small in the area	Lattice Wilson loops; the relation to local observables is not treated
Upper bound on the vortex free energy, thin-vortex floor, thick-vortex floor, pairs of Polyakov loops, the vortex ratio	4 (another way of stating confinement)	Twisted boundary conditions and centre vortices Source (not found in the published construction, within the range of a search by name)	The rate is independent of the transverse size only inside the strong-coupling window (the prefactor remains)

Item of this book's study	Open question of the same kind	What is the same	What differs
Rarity of large plaquettes	No direct relation	—	A one-scale estimate of the type that controls large fields
Fermion weight	(Outside pure Yang–Mills)	—	Only properties of the vessel. Nothing about a mass gap or the continuum limit

"Of the same kind" in Table 1 means that we hold the same kind of question at the opposite end of the lattice. It does not mean that a theorem of this book's study answers an open question of the published construction, or points towards one.

09

A bridge to Volume 3

This section is an outline, and includes inference.

Where the published construction stops. The published construction carries every step at weak coupling, $\beta_m \in [\hat{\beta}_m/2, 2\hat{\beta}_m]$, and stops at the step with lattice spacing 1 (step 0) Source . The coupling at step 0 is the large reference value $\bar{\beta}$, and the volume L_p is measured in that unit Source . For distances longer than spacing 1, no statements of exponential decay of correlations, an area law or uniqueness of the infinite-volume limit were found within what we read Source (within the range of a search by name). What there is are the polynomial upper bounds of § 08. The lengths at which confinement and a mass gap become visible lie further beyond Inferred .

A candidate restatement of the remaining questions. One conceivable route is to restate the remaining questions (mass gap, area law, uniqueness) as the question of "following the effective theory at step 0 (with large coupling $\bar{\beta}$) to long distances, from weak coupling to strong coupling" Inferred . What this book's study has called "the crossover band" (Part 26) lies in between. However, this cannot be asserted. The theory at step 0 is not a lattice theory with the Wilson action but an effective action with a representation (clipped action, activities, gas), and the continuum correlations are not a function of the step-0 measure alone Source .

Statements missing for the restatement. Putting this restatement in the form of statements needs at least five statements not found in the published construction Source (within the range of a search by name).

1. Existence and a name for the limiting object, as the cutoff $N \to \infty$, of the push-forward densities at step 0 (uniform Cauchy for the densities exists: `Flux.fanned`;

the limit would presumably follow Inferred , but there is no named statement).

2. Reflection positivity of the push-forward measure at step 0 (or at any step) (not trivial because the patches of the kernel straddle the plane Inferred). In this book's design, `Block3.rp_push` has a corresponding statement for this book's map, but not for the published construction's kernel.
3. The relation between Wilson loops of coarse configurations and Wilson loops of the fine (or continuum) field.
4. Local estimates at step 0 uniform in the volume (estimates at the level of densities for step-0 objects contain exponentials of the volume Source).
5. The images of observables at step 0, and their limits.

Where this book's holdings stand. Many of the Lean theorems of this book's study lie at the strong-coupling end of this gap (the mass gap, clustering, the area law, and the vortex free energy whose rate is independent of the transverse size inside the strong-coupling window). What holds at every coupling is the thin-vortex floor, whose rate depends on the transverse size. The gap in between remains open. Volume 3 starts from the infrared edge of the published construction (16-03) and this book's position (16-04), and writes about this gap in the form of conditional statements and the tree of premises (Parts 27 and 28).

What this chapter can say

Can say	Cannot say
The Lean of this book's study: in strong-coupling windows, an infinite-volume limit without subsequences, a mass gap (with non-degeneracy), clustering (Part 20). Formalisations of known types	Outside the windows, varying the lattice spacing, a mass gap in the continuum limit
The Lean of this book's study: the strong-coupling area law (finite box and infinite volume), an upper bound on the vortex free energy (inside the strong-coupling window, the rate per area does not depend on N, M; the prefactor N^2 remains). Formalisations of known types	Uniform forms outside the window, the sign side $Z_1 \leq Z_0$, weak coupling
The Lean of this book's study: the thin-vortex floor (all real β), the thick-vortex floor, pairs of Polyakov loops, continuity and matching of the vortex ratio (Part 21)	Rates uniform in the transverse size outside the strong-coupling window, confinement, the continuum limit
The Lean of this book's study: rarity of large plaquettes at weak coupling (β large), uniform in volume, explicit threshold (22-01)	Measures at coarse scales, a form spanning steps, a form uniform in the lattice spacing, uniqueness

Can say	Cannot say
The Lean of this book's study: three items specific to the design (consistency of push-forward by a fixed block map, reflection positivity of the pushed-forward measure, matching by quantities determined by the measure)	All are statements at the level of design and say nothing about the content of the continuum limit
Source (within the range of a search by name) No statement of exponential decay of long-distance correlations, no continuum Wilson loops and no boundary conditions other than periodic were found in the published construction. What there is at long distances are polynomial upper bounds uniform over the phases	We have not searched the whole repository other than by name
Inferred One conceivable route is to restate the remaining questions as "long-distance questions for the effective theory at step 0"	At least five statements needed for that restatement are missing (§ 09). The theory at step 0 is not the Wilson action

Sources and reproduction

Item	Kind	Source / tool
Theorems for the strong-coupling lattice	Lean	The tables of sources in 20-02, 20-03, 20-04 (`InfVol.wilson_infvol_unique`, `WilsonLimit.cluster_limit`, `WilsonLimit.wilson_massGap_nondegenerate`, `WilsonLimit.wilson_massGap_dob_exists`, `InfMain2.wilson_infvol_main2` and others)
Strong-coupling area law	Lean	The Lean bundle of this book's study (`AreaLaw.area_law_wilson`, `area_law_wilson_area`, `area_law_wilson_general`, `area_law_wilson_sharp`, `area_law_wilson_max_sharp`, `area_law_infvol`, `area_law_limit_sub`). Passed the kernel recheck and checked in the source. Details in 20-06
Upper bound on the vortex free energy	Lean	`VortexStrong.vortex_floor`, `VortexStrong.vortex_floor_sharp` (isotropic torus), `VortexStrongMN.vortex_floor_MN_sharp` (width N, height M). 20-06
Theorems on centre vortices	Lean	The tables of sources in 21-02, 21-03, 21-05, 21-06 and 21-07 (`VortexCert.SN_two_lt`, `SN_three_lt`, `SN_four_lt` are computer certificates)
Rarity of large plaquettes	Lean	The table of sources in 22-01
Fermion weight	Lean	The table of sources in 20-05
Three items specific to the design	Lean	`ChainBlockMap.hcons_blockMap` (19-04), `Block3.rp_push`, `Block3.tuned3_exists` (14-02, 14-03), `ThetaLimit.tuning_all` (21-07)

Item	Kind	Source / tool
Known types of the area law, strong coupling and vortex free energy	Cited	Osterwalder–Seiler (strong-coupling area law), the Dobrushin condition, 't Hooft's twisted boundary conditions and vortex free energy. Bibliographic details in the tables of sources of each chapter and in A-03
Open questions	Assessment	The assessment (`docs/assessment-1.pdf` in `yang-mills-group/YM`, 2026-11-10), § 8
The infrared side of the published construction, and step 0	Source	In the repository `yang-mills-group/OpenAI-YM-20261028` (Apache-2.0): `YM/EveryPhase/Light.lean:221` (`light_bound`), `YM/EveryPhase/Decay.lean:173` (`inverse_square`), `YM/Lattice/Path.lean:105` (`travel`), `YM/Wilson/Law.lean:87` (`law`, periodic torus), `YM/Slice/Reflection.lean:98` (`reflected_positive`), `YM/RG/History.lean:213` (field `band` of `Conv`), `YM/FullVolume/Fields/Cement.lean:50` (`fanned`)

Terms used in this chapter — strong-coupling window, infinite-volume limit, mass gap, clustering, area law, Wilson loop, twisted partition function, vortex free energy, thin-vortex floor, thick-vortex floor, Polyakov loop, vortex ratio, rarity of large plaquettes, fermion weight, push-forward measure, crossover band. Their meanings are in the A-02 glossary.

Next chapter: 15-00 Entrance — what ultraviolet control is — the entrance to the last part of Volume 2 (Part 15, Record of the attempts at ultraviolet control). To go on to Volume 3 first, 16-02 Five open questions — we read the open questions of the published construction one by one.